ALTERNATING CURRENT

PRACTICE QUESTION (Pg No. 9)

1) A light bulb is rated 150 W for 220 V AC supply of 60 Hz. Calculate
   (a) **the resistance of the bulb**
   (b) **the rms current through the bulb.** **[All India 2012]**

SOL: (a) *P =150* W, V = 220 V,

Resistance of the bulb $R = \frac{V^2}{P} = \frac{220 \times 220}{150} = 322.7\Omega$

(b) $I_{rms} = \frac{V_{rms}}{R} = \frac{220}{322.7} \quad (V_{rms} = V = 220 \text{ V})$

$I_{rms} = 0.68\ \Omega$

2) Distinguish between the terms 'average value' and 'rms value' of an alternating current. The instantaneous current from an AC source is I = 5 sin 314t A. What are the average and rms values of the current? [Delhi 2007][All India 2010C]

SOL: The rms value of alternating current is equal to that value of DC(Steady current) which produces same amount of heat in a given resistance as produced by the given AC, when passed for the same time (T). $I_{rms} = \frac{I_0}{\sqrt{2}}$

where, I_0 = peak value of AC.

Average value: The average or mean value of AC is equal to that direct current which sends the same charge in a circuit in the same time as is sent by the given AC in the same circuit in its half time period. $I_{av} = \frac{2}{\pi} I_0 = 0.637\ I_0$

Given I = 5 sin 314t A

Comparing with $I = I_0 \sin \omega t$

$I_0 = 5$,

$I_{av} = \frac{2}{\pi} I_0 = 0.637\ I_0 = 0.637 \times 5 = 3.185A$

$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 0.707 \times 5 = 3.353A$

3) Distinguish between the terms 'effective value' and 'peak value' of an alternating current. An alternating current from a source is represented by I =10 sin 314t A. Write the corresponding values of
(a) **its 'effective value'.**
(b) **frequency of the source. [Delhi 2007]**

SOL: Effective value Effective value of alternating current is also known as rms value for which is as **The rms value** of alternating current is equal to that value of DC which produces same amount of heat in a given resistance as produced by the given AC, when passed for the same time (T).

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$I_{rms} = \frac{I_0}{\sqrt{2}}$ where, I_0 = peak value of AC.

Peak value The maximum value of an alternating current is termed as peak value of an AC.

(a) Given, $I = 10 \sin(314t)$

Comparing with

$$I = I_0 \sin(\omega t)$$

$$\Rightarrow \quad I_0 = 10 \text{ A}$$

$$\omega = 314$$

rms value $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{10}{\sqrt{2}}$ A

Effective value $= I_{rms} = \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 5\sqrt{2}$ A

(b) $\because \quad \omega = 314$

$\therefore \quad 2\pi f = 314$

where f = frequency of AC.

$$f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = \frac{314}{2 \times 314} \times 100 = 50$$

$$f = 50 \text{ Hz}$$

4) The current flowing through a pure inductance 2 mH is I = 15 cos 300t A. What is the (a) rms and (b) average value of current for a complete cycle? [Foreign 2011]

SOL: Current flowing through the inductor $I = 15 \cos(300t)$

Comparing with $I = I_0 \sin \omega t$

Here, peak value of current $I_0 = 15$ A

$$\text{(a)} \quad I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{15}{\sqrt{2}} \text{ A}$$

(b) For complete cycle, average value of current is zero $I_{av} = 0$

5) An AC current, $I = I_0 \sin\omega t$, produces a certain heat *H*, in a resistor *R*, over a time $T = \frac{2\pi}{\omega}$. Write the value of the DC current that would produce the same heat, in the same resistor, in the same time. [All India 2009C]

SOL: The required current as asked in the question is i_{rms} and given by $I_{rms} = \frac{I_0}{\sqrt{2}}$

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PRACTICE QUESTION (Pg. No 11)

Example-5

The electric main in a house is marked 220V-50 Hz. Write down the equation for instantaneous voltage. **Ans: V = 311 sin 100 πt**

SOL: The equation for the alternating voltage is

$$V = V_0 \sin \omega t = V_0 \sin 2\pi f t,$$

where V is instantaneous voltage and V_0 the peak value of the voltage.

Here rms (virtual) voltage is $V_{rms} = 220$ V.

$$\therefore \; V_0 = \sqrt{2}\, V_{rms} = 1{\cdot}414 \times 220 \text{ V} = 311 \text{ V}.$$

Also $f = 50$ Hz. Substituting values of V_0 and f in eq. (i), we get

$$\boldsymbol{V = 311 \sin 100\, \pi t}.$$

PRACTICE QUESTION (Pg. No. 13)

1) **A 100-Ω resistor is connected to 220 V - 50 Hz a.c. supply. Find rms value of current in the circuit and the net power consumed for a complete cycle.**

SOL: Given : V_{rms} = 220 V .

$$\therefore \; i_{rms} = \frac{V_{rms}}{R} = \frac{220 \text{ V}}{100\ \Omega} = \mathbf{2{\cdot}20\ A}.$$

The average power consumed for a complete cycle is $\overline{P} = i_{rms}^2 R = (2.20A)^2 \times 100\Omega = 484W$

2) **A sinusoidal voltage V = 200 sin 314 *t* is applied to a 10Ω resistor. Find (a) the frequency of the supply, (b) peak voltage, (c) rms voltage, (d) rms current and (e) power dissipated as heat.**

SOL:

Comparing the given voltage equation with $V = V_0 \sin \omega t$, we have

$$V_0 = 200 \text{ V} \quad \text{and} \quad \omega = 314 \text{ s}^{-1}.$$

(a) supply frequency, $f = \frac{\omega}{2\pi} = \frac{314 \text{ s}^{-1}}{2 \times 3{\cdot}14} = \mathbf{50\ Hz}$. **(b)** Peak voltage, $V_0 = \mathbf{200\ V}$.

(c) rms voltage, $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{200 \text{ V}}{1{\cdot}414} = \mathbf{141{\cdot}4\ V}$.

(d) rms current, $i_{rms} = \frac{V_{rms}}{R} = \frac{141{\cdot}4 \text{ V}}{10\ \Omega} = \mathbf{14{\cdot}14\ A}$. **(e)** The power is dissipated in resistance only.

$$\therefore \; P = (i_{rms})^2 R = (14{\cdot}14 \text{ A})^2 \times 10\ \Omega = \mathbf{2000\ W}.$$

3) **An alternating voltage given by V = 280 sin 50π t is connected across a pure resistor of 40Ω. Find**
   - **a) the frequency of the source.**
   - **b) the rms current through the resistor. [All India 2012]**

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SOL: V = 280 sin 50π t , R= 40Ω,

Comparing with $V = V_0 \sin \omega t$, $V_0 = 280$

(a) ω = 50 π rad/s

$2\pi f = 50\pi \qquad (\because \omega = 2\pi f)$

$f = \frac{50\pi}{2\pi} = 25Hz$

(b) $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{280}{\sqrt{2}}$ V

$\therefore \quad I_{rms} = \frac{V_{rms}}{R} = \frac{280}{\sqrt{2} \times 40}$

$= \frac{280}{1.414 \times 40} = 4.95$ A

4) An alternating voltage given by V = 70 sin 100π t is connected across a pure resistor of 25 Ω. Find

a) the frequency of the source.

b) the rms current through the resistor. [All India 2012]

SOL: V = 70 sin 100π t

Comparing with $V = V_0 \sin \omega t$, $V_0 = 70$

(a) ω = 100 π rad/s

$2\pi f = 100\pi \qquad (\because \omega = 2\pi f)$

$f = \frac{100\pi}{2\pi} = 50Hz$

(b) $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{70}{\sqrt{2}}$

$I_{rms} = \frac{V_{rms}}{R}$

$= \frac{(70/\sqrt{2})}{25}$

$= \frac{70}{1.414 \times 25}$

= 1.98 A

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PRACTICE QUESTIONS (Pg. No.29)

1) **An electric bulb is designed to consume 55 W when operated at 110 volt. It is connected to a 220 V, 50 Hz line through a choke coil in series. What should be the inductance of the coil for which the bulb gets correct voltage?**

SOL: 55 W and 110V are DC values

$$R = \frac{V^2}{P} = \frac{110 \times 110}{55}\ \Omega = 220\ \Omega$$

$$\text{Current through the bulb} = \frac{55}{110}\ A = \frac{1}{2}\ A$$ **in DC supply.**

In AC :

$$\text{Now,} \quad \frac{1}{2} = \frac{220}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\text{or} \quad R^2 + \omega^2 L^2 = 440 \times 440$$

$$\text{or} \quad L^2 \times 10^4 \pi^2 = 440 \times 440 - 220 \times 220$$

$$\text{or} \quad L^2 = \frac{220 \times 220 \times 3}{10^4 \times \pi^2}$$

$$\text{or} \quad L = \frac{220}{100 \times 3.14}\sqrt{3}\ H = \mathbf{1.2\ H}$$

2) **Can a capacitor of suitable capacitance replace a choke coil in an a.c. circuit ?**

Ans. Yes, because the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in a.c. circuit without power dissipation.

3) **(i) A choke coil and a bulb are connected in series to a d.c. source. If an iron core is inserted in the choke coil, is there any change in the brightness of the bulb ?**
(ii) What if the choke coil and the bulb are connected to an a.c. source ?

Ans: No, the reactance X_L *(=* ω *L=2πf L)* of the choke coil is zero for d.c. Hence any change in *L* does not effect d.c. and the bulb continues to shine as brightly as before.

(ii)In this case, the brightness of the bulb decreases because the choke coil offers reactance ωL to a.c. When an iron core is inserted in the coil, the inductance *L* of the coil and hence its reactance *of L* increases. Therefore, the current in the bulb further decreases and now it shines with less brightness.

4) **An ac voltage of 100 V, 50 Hz is connected across a 20 Ω resistor and 2 mH inductor in series. Calculate (i) impedance of the circuit (ii) rms current in the circuit.[CBSE 2007]**

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SOL: (i) Z= $\sqrt{R^2+\omega^2 L^2}=\sqrt{R^2+4\pi^2 f^2 L^2}$

or $\quad Z = \sqrt{400 + 4 \times 9.88 \times 2500 \times (2 \times 10^{-3})^2}\ \Omega$

$= \mathbf{20.01\ \Omega}$

(ii) $I_{rms}=\dfrac{V_{rms}}{Z}=\dfrac{100}{20.01}A\approx 5A$

5) **When an inductor L and a resistor R in series are connected across a 12 V, 50 Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by π/3 radian. Calculate the value of R. [CBSE 2006]**

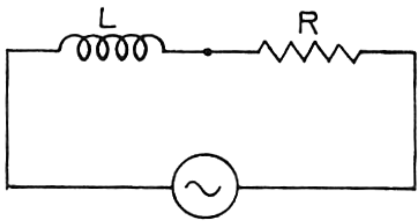
SOL:

$$\text{Impedance, } Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{0.5} = 24\ \Omega$$

As $\quad \cos\phi = \dfrac{R}{Z}$

$\therefore \quad R = Z\cos\phi = 24\cos\dfrac{\pi}{3} = 24 \times \dfrac{1}{2} = \mathbf{12\ \Omega}$

6) **In the given circuit, the potential difference across the inductor L and resistor R are 200 V and 150 V respectively and the rms value of current is 5 A. Calculate (i) the impedance of the circuit and (ii) the phase angle between the voltage and the current. [CBSE 2004]**



SOL: V_L = 200 V, V_R = 150 V, I_{rms} = 5 A

$V_{rms}=\sqrt{V_L^2+V_R^2}=\sqrt{200^2+150^2}=250V$

(i) $I_{rms}=\dfrac{V_{rms}}{Z}; Z=\dfrac{V_{rms}}{I_{rms}}=\dfrac{250}{5}=50\Omega$

$(ii)\ \tan\phi = \dfrac{V_L}{V_R} = \dfrac{200}{150} = 1.3333$

$\phi = \tan^{-1}(1.3333) = \mathbf{53.13^\circ}$

7) **A 100 V, 50 Hz ac source is connected to a series combination of an inductance of 100 mH and a resistance of 25 Ω Calculate the magnitude and phase of the current.[CBSE 1991]**

SOL:

$X_L = \omega L = 2 \times 3.14 \times 50 \times 100 \times 10^{-3}\ \Omega$

$= 31.4\ \Omega$

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$$Z = \sqrt{R^2 + {X_L}^2} = \sqrt{25^2 + 314^2} = 40.14\Omega$$

$$I_{rms} = \frac{100}{40.14} = 2.49A$$

$$\tan\phi = \frac{X_L}{R} = \frac{31.4}{25} = 1.256,$$

$$\phi = \tan^{-1}(1.256) = \mathbf{51.47}°.$$

8) A bulb of resistance 10 Ω connected to an inductor of inductance L is in series with an ac source marked 100 V, 50 Hz. If the phase angle between voltage and current is $\frac{\pi}{4}$ radian, calculate the value of L. [CBSE]

SOL:

$$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f\,L}{R}$$

$$L = \frac{R\tan\phi}{2\pi f}$$

Now, $\phi = 45°$, $\tan\phi = 1$, $f = 50$ Hz, $R = 10\ \Omega$

$$\therefore \quad L = \frac{10 \times 1 \times 7}{2 \times 22 \times 50}\ \text{H} = \mathbf{0.032\ H}$$

9) An inductance coil has a reactance of 100 Ω. When ac signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. Calculate the self-inductance of the coil. [CBSE 1998 S]

Ans. It is clear from the given data that the inductance is not pure inductance. The given coil behaves as a series combination of L and R.

$$\tan\phi = \frac{X_L}{R},\ \tan 45° = \frac{X_L}{R} \quad \text{or} \quad R = X_L$$

Now, $$Z = \sqrt{R^2 + {X_L}^2}$$

or $$100 = \sqrt{{X_L}^2 + {X_L}^2} = \sqrt{2}\,X_L$$

or $$X_L = \frac{100}{\sqrt{2}}\ \Omega = 70.7\ \Omega$$

or $$\omega L = 70.7 \quad \text{or} \quad 2\pi f L = 70.7$$

or $$L = \frac{70.7 \times 7}{2 \times 22 \times 1000}\ \text{H} = \mathbf{1.12 \times 10^{-2}\ H.}$$

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10) An AC voltage of 100 V, 20 Hz is connected across a 20 Ω resistor and 2 mH inductor in series. Calculate (a) impedance of the circuit, (b) rms current in the circuit. [All India 2007]

SOL:

$V_{rms} = 100$ V, $f = 20$ Hz

$R = 20\ \Omega$, $L = 2 \times 10^{-3}$ H

$X_L = 2\pi f L = 2\pi \times 20 \times 2 \times 10^{-3}$ H

(a) Impedance of the circuit $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{(20)^2 + (80\pi \times 10^{-3})^2}$$

$$= 20 \times \sqrt{1 + (4\pi \times 10^{-3})^2}$$

Impedance $Z \approx 20\ \Omega$ nearly

(b) rms current $I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{20} = 5$ A

11) A 60-V, 10-W lamp is to be used on a.c. source of 100 V, 60 Hz. Calculate the inductance of the choke coil required to be put in series to run the lamp. How much pure resistance should be used in place of the choke coil, so that the lamp may run on its rated voltage ?

SOL: 60-V, 10-W is DC values

The current taken by lamp is $i = \frac{P}{V} = \frac{10W}{60V} = \frac{1}{6} A$

and the resistance of its filament is

$$R = \frac{V}{i} = \frac{60\text{ V}}{(1/6)\text{ A}} = 360\ \Omega .$$

If the lamp is to run on 100 V-60 Hz a.c. mains, a choke-coil should be placed in series with it so that its effective resistance (impedance) is increased and the current through it does not exceed (1/6) A. Let L be the inductance of the required choke. Then, the impedance of the lamp-choke combination is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f)^2 L^2}$$

$$= \sqrt{(360)^2 + (2 \times 3{\cdot}14 \times 60)^2 L^2} = \sqrt{(360)^2 + (377\, L)^2} .$$

Now, current = voltage/impedance . Thus,

$$\frac{1}{6} = \frac{100}{\sqrt{(360)^2 + (377\, L)^2}}$$

$$(360)^2 + (377\, L)^2 = (600)^2$$

$$(377\, L)^2 = (600)^2 - (360)^2 = 230400$$

$$377\, L = 480$$

$$L = \frac{480}{377} = \mathbf{1{\cdot}27\ H.}$$

Let R’ be the resistance of the circuit for which the current in the lamp would be (1/6)A. Then

$$i_{rms} = \frac{V_{rms}}{R'}$$

or $$\frac{1}{6} = \frac{100}{R'} \quad \text{or} \quad R' = 600\,\Omega .$$

The resistance R of the lamp itself is 360 Ω. Therefore, the additional resistance required for the circuit is

$$R' - R = 600 - 360 = \mathbf{240\,\Omega} .$$

This will, however, result in waste of electrical energy as heat in the additional resistance.

12) A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5Ω when running at 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated at 160 V (dc), what additional resistance is required. Compare the power losses in both cases.

SOL: The current (rms) in an L - R circuit is given by

$$i_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} .$$

Here $i_{rms} = 10$ A, $V_{rms} = 160$ V, $R = 5\,\Omega$ and $\omega = 2\pi f = 2 \times 3{\cdot}14 \times 50 = 314\ \text{s}^{-1}$.

$$\therefore \quad 10 = \frac{160}{\sqrt{(5)^2 + (314\,L)^2}}$$

or $$(314\,L)^2 = (16)^2 - (5)^2 = 231$$

or $$L = \frac{\sqrt{231}}{314} = \frac{15{\cdot}2}{314} = \mathbf{4{\cdot}84 \times 10^{-2}\ H} .$$

Let R_A be the additional resistance required in dc circuit. Then

$$i_{dc} = \frac{V_{dc}}{R + R_A} .$$

or $$10\ \text{A} = \frac{160\ \text{V}}{5\,\Omega + R_A}$$

or $$R_A = \mathbf{11\,\Omega} .$$

The power loss in ac circuit is given by

$$P_{ac} = V_{rms} \times i_{rms} \times \cos\phi .$$

For LR circuit, $$\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}} = \frac{5\,\Omega}{\sqrt{(5\,\Omega)^2 + \left(314\ \text{s}^{-1} \times \frac{15{\cdot}2}{314}\ \text{H}\right)^2}} = \frac{5}{16} .$$

$$\therefore P_{ac} = 160\ \text{V} \times 10\ \text{A} \times \frac{5}{16} = \mathbf{500\ W} .$$

The power loss in dc circuit is

$$P_{dc} = V_{dc} \times i_{dc} = 160\ \text{V} \times 10\ \text{A} = 1600\ \text{W} .$$

Thus $$P_{ac} : P_{dc} = 500 : 1600 = \mathbf{5 : 16} .$$

13) A student connects a long air-cored coil of manganin wire to a 100-V d.c. source and records a current of 1.5 A. When the same coil is connected across 100 V-50 Hz a.c. source, the current reduces to 1.0 A. Why ? Calculate the reactance of the coil.

SOL: For DC , we have $i_{dc} = \frac{V}{R}$ ...(i)

While in A , we have $i_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_L^2}}$ ....(ii)

Where X_L is the reactance of the coil . Here V = 100 V and also V_{rms} = 100V

$\therefore i_{rms} = i_{dc}$

From eq. (i) , we have $R = \frac{V}{i_{dc}} = \frac{100V}{1.5A} = 66.67\Omega$

From eq. (ii) , we have

$$X_L{}^2 = \frac{V_{rms}^2}{i_{rms}^2} - R^2 = \frac{(100V)^2}{(1.5A)^2} - (66.67)^2 = (100)^2 - (66.67)^2 = 5555\Omega^2$$

$$X_L = 74.53\Omega$$

14) A virtual current of 4 A flows in a coil when it is connected in a circuit having a.c. of frequency 50 Hz. The power consumed in the coil is 240 W. Calculate the inductance of the coil if the virtual p.d . across it is 100 V.

SOL: Let L be the inductance, R the resistance and Z the impedance of the coil. The power is consumed in R only. Thus, the average power is $\overline{P} = (i_{rms})^2 R$

$$R = \frac{\overline{P}}{(i_{rms})^2} = \frac{240}{4^2} = 15\Omega$$

Now, $$Z = \frac{V_{rms}}{i_{rms}} = \frac{100 \text{ V}}{4 \text{ A}} = 25\ \Omega.$$

Again, $$Z = \sqrt{R^2 + (\omega L)^2}$$

or $$(\omega L)^2 = Z^2 - R^2 = (25\ \Omega)^2 - (15\ \Omega)^2 = 400\ \Omega^2$$

or $$\omega L = 20\ \Omega.$$

Here $\omega = 2\pi f = 2 \times 3{\cdot}14 \times 50 = 314 \text{ rad s}^{-1}$.

$$\therefore \quad L = \frac{20\ \Omega}{\omega} = \frac{20\ \Omega}{314 \text{ rad s}^{-1}} = \mathbf{0{\cdot}0637\ H}.$$

15) An a.c. circuit having an inductor and a resistor in series draws a power of 560 W from an a.c. source marked 210 V-60 Hz. The power factor of the circuit is 0.8. Calculate the impedance of the circuit and the inductance of the inductor.

SOL: The average power over complete cycle is given by $\overline{P} = V_{rms} \times I_{rms} \times \cos\phi$, where $\cos\phi$ is power factor

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$$i_{rms} = \frac{\overline{P}}{V_{rms} \times \cos\phi} = \frac{560W}{210V \times 0.8} = \frac{10}{3}A$$

The impedence of the circuit $Z = \frac{V_{rms}}{i_{rms}} = 63\Omega$

The power is consumed in R only. Therefore

$$P = (i_{rms})^2 R$$

or $$R = \frac{P}{(i_{rms})^2} = \frac{560\text{ W}}{\left(\frac{10}{3}\text{ A}\right)^2} = 50{\cdot}4\ \Omega .$$

Now, the impedance of an $L - R$ circuit is

$$Z = \sqrt{R^2 + (\omega L)^2} .$$

$$\therefore\ (\omega L)^2 = Z^2 - R^2 = (63\ \Omega)^2 - (50{\cdot}4\ \Omega)^2$$
$$= (63 + 50{\cdot}4)\ \Omega \times (63 - 50{\cdot}4)\ \Omega$$
$$= 113{\cdot}4\ \Omega \times 12{\cdot}6\ \Omega = 1428{\cdot}84\ \Omega^2$$

or $$\omega L = \sqrt{1428{\cdot}84} = 37{\cdot}8\ \Omega .$$

$$\therefore\ L = \frac{37{\cdot}8\ \Omega}{2\pi f} = \frac{37{\cdot}8\ \Omega}{2 \times 3{\cdot}14 \times 60\text{ s}^{-1}} = \mathbf{0{\cdot}1\ H.}$$

16) An alternating voltage E = 200 sin 300t is applied across a series combination of R = 10Ω and L = 800 mH. Calculate (i) the impedance of the circuit (ii) the peak current in the circuit and (iii) the power factor of the circuit. [CBSE 1994]

Ans. Comparing the given equation with $E = E_0 \sin \omega t$, we get ω = 300 rad s^{-1}.

Now, $X_L = \omega L = 300 \times 800 \times 10^{-3}\ \Omega = 240\ \Omega$

(i) Impedence $Z = \sqrt{R^2 + {X_L}^2} = \sqrt{10^2 + 240^2} = 240.2\Omega$

(ii)Peak current , $I_0 = \frac{V_0}{Z} = \frac{200}{240.2} = 0.83A$

(*iii*) Power factor, $\cos\phi = \frac{R}{Z} = \frac{10}{240.2} = \mathbf{0.0416.}$

17) When 100 volt dc is applied across a coil, a current of one ampere flows through it. When 100 volt ac of 50 cycle per second is applied to the same coil, only 0.5 ampere flows. Calculate (i) resistance of coil (ii) impedance of coil (iii) inductive reactance of coil (iv) inductance of coil. [CBSE 1997]

Ans: (i) V = 100 V, I = 1 A, R = ?

Ohmic resistance , $R = \frac{V}{I} = \frac{100V}{1A} = 100\Omega$

(ii) V_{rms} = 100 V , f = 50 s^{-1} , , I_{rms} = 0.5 A

Impedence of coil $Z = \frac{V_{rms}}{I_{rms}} = \frac{100V}{0.5A} = 200\Omega$

(iii) $Z^2 = R^2 + X_L^2; X_L = \sqrt{Z^2 - R^2} = \sqrt{200^2 - 100^2} = 100 \times 1.732 = 173.2\Omega$

(iv) $X_L = 2\pi f L$

$$L = \frac{X_L}{2\pi f} = \frac{173.2}{2 \times 3.14 \times 50} = \mathbf{0.55\ H.}$$

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PRACTICE QUESTIONS (Pg . No. 36)

7) **A 100 μF capacitor in series with a 40Ω resistance is connected to a 100 V, 60 Hz supply. Calculate (i) the reactance (ii) the impedance, and (iii) maximum current in the circuit.[CBSE 2005 S]**

SOL: (i) Reactance $X_C = \frac{1}{2\pi f\ C}$

$$= \frac{7}{2 \times 22 \times 60 \times 100 \times 10^{-6}}\ \Omega = \mathbf{26.5\ \Omega}$$

(ii) Impecdence Z = $\sqrt{R^2 + X_C^2}$

$$= \sqrt{40^2 + 26.5^2}\ \Omega = \mathbf{47.98\ \Omega}$$

$$(iii) I_0 = \frac{V_0}{Z} = \frac{\sqrt{2}V_{rms}}{Z} = \frac{1.414 \times 100}{47.98} A = 2.95A$$

8) **An alternating current of 1.5 mA and angular frequency 300 radian s^{-1} flows through a 10 lent resistor and a 0.50 μF capacitor in series. Find the rms voltage across the capacitor and impedance of the circuit. [CBSE 1993]**

SOL: $X_C = \frac{1}{\omega C} = \frac{1}{300 \times 0.50 \times 10^{-6}} = \frac{10^6}{150} = \frac{1000}{150} \times 10^3 = \frac{20}{3} \times 10^3 \Omega$

Impedence, Z = $\sqrt{R^2 + X_C^2} = \sqrt{10^8 \times \frac{4}{9} \times 10^8} = 1.2 \times 10^4 \Omega$

Rms voltage across the capacitor = $I_{rms} . X_C = 1.5 \times 10^{-3} \times \frac{20}{3} \times 10^3 = 10V$

9) **A 100 μF capacitor is in series with a 40Ω resistor and is connected to a 100 V, 50 Hz AC source. Calculate the following**
   a) Maximum current in the circuit.
   b) Time lag between current maximum and voltage maximum. [Foreign 2008]

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SOL: Given , V_{rms} =100V , C = 100μF = 100×10^{-6} F , R = 40 Ω

$$\text{Total impedance of the circuit} = \sqrt{R^2 + X_C^2} = \sqrt{(R)^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{(40)^2 + \left(\frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}}\right)^2}$$

$$\simeq 51.13\ \Omega$$

Current in the circuit $I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{51.12} = 1.95A$

Maximum Current $I_0 = I_{rms}\sqrt{2} = 1.95\sqrt{2} = 2.76A$

(b) For pure capacitor, phase difference $\phi = \frac{\pi}{2}$

Time difference = $\frac{T}{2\pi}\times$ Phase difference = $\frac{\frac{1}{50}}{2\pi}\times\frac{\pi}{2} = \frac{1}{200}s$

Time lag between current maximum and voltage maximum = $\frac{1}{200}s$.

10) A resistor of 200 Ω and a capacitor of 40 μF are connected in series to 220 V AC source with angular frequency (ω) = 300 Hz. Calculate the voltages (rms) across the resistor and the capacitor. Why is the algebraic sum of these voltages more than the source voltage? How do you resolve this paradox?[Foreign 2007]

SOL:

$R = 200\ \Omega$, $C = 40\ \mu F$, $V_{rms} = 220$ V,
$\omega = 300$ Hz

Inductive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{300 \times 40 \times 10^{-6}}$$

$$= 83.33\ \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{(200)^2 + (83.33)^2}$$

$$= 216.6\ \Omega \text{ (nearly)}$$

$$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{216.6} = 1.02\text{ A}$$

Rms voltage across the R is

$$V_R = I_{rms} R = 1.02 \times 200 = 204V$$

$$Across\ capacitor\ V_C = I_{rms} X_C = 1.02 \times 83.33 = 85V$$

$$V_{rms} \neq V_R + V_C$$

Because V_C and V_R are not in same phase, there is why

$\because V_R$ leads V_C by phase $\frac{\pi}{2}$

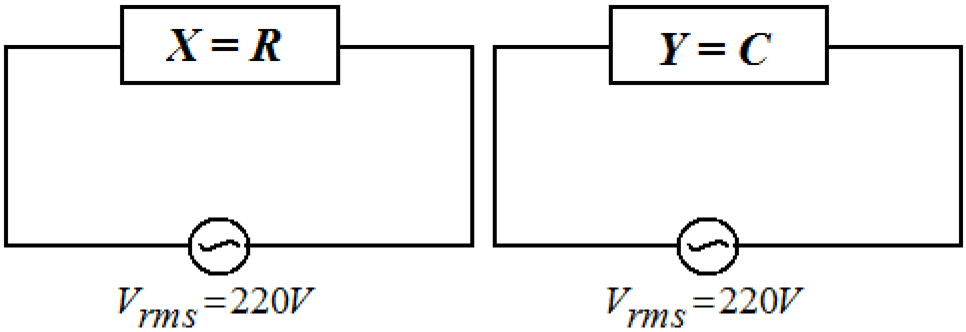
$$V_{rms} = \sqrt{V_R^2 + V_C^2}$$

11) When an alternating voltage of 220 V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, again the same current flows but it leads the voltage by $\pi/2$. (a) Name the devices X and Y. (b) Compute the current in the circuit when the same voltage in applied across the series combination of X and Y. [CBSE]

SOL:

(**a**) X is a **resistor** and Y is a **capacitor.**

(**b**) The resistance of X and the reactance of Y are



$$R = \frac{V_{rms}}{I_{rms}} = \frac{220}{0.5} = 440\Omega$$

$$R = X_C = 440\Omega$$

When R and C are in series, then

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_C)^2}}$$

$$= \frac{220\ \text{V}}{\sqrt{(440\ \Omega)^2 + (440\ \Omega)^2}} = \frac{220\ \text{V}}{440\sqrt{2}\ \Omega} = \mathbf{0{\cdot}3536\ A}.$$

PRACTICE QUESTIONS (Pg.No. 43)

Example 5:

An inductor 200 mH, a capacitor C and a resistor 10 Ω are connected in series with a 100 V, 50s^{-1}AC source. If the current and voltage are in phase with each other, calculate the capacitance of the capacitor. [All India 2006C] Ans: 50 μF

SOL: Current and voltage are in same phase it implies that the circuit is in resonance.

$\therefore$ Resonating frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{4 \times \pi^2 \times (50)^2 \times 2 \times 10^{-1}}$$

$$= 50 \times 10^{-6} \text{ F (nearly)}$$

$$= 50\ \mu\text{F}$$

PRACTICE QUESTION (Pg. No. -75)

1) Show that power dissipated at resonance in L-C-R circuit is maximum.

SOL:

The power dissipated in an L-C-R circuit is given by

$$\overline{P} = V_{rms} \times i_{rms} \times \cos\phi .$$

The power factor $\cos\phi \left(= \dfrac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} \right)$ is maximum at resonance (= 1). Hence the power dissipated is maximum at resonance.

2) An AC source, of voltage V = V_0 sin ωt, is applied across a

(a) series *RC* circuit in which the capacitative impedance is a times the resistance in the circuit.

(b) series *RL* circuit in which the inductive impedance is *b* times the resistance in the circuit.

Calculate the value of the power factor of the circuit in each case. [All India 2008C]

SOL:

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$\because$ Power factor $\cos\phi = \dfrac{R}{Z}$

where, ϕ = phase difference between V and Z

R = ohmic resistance

Z = impedance.

(a) Given, $X_C = aR$

$\therefore$ Impedance $Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{R^2 + a^2R^2} = R\sqrt{1 + a^2}$$

$$\therefore \quad \cos\phi = \frac{R}{R\sqrt{1+a^2}} = \frac{1}{\sqrt{1+a^2}}$$

$$\text{Power factor} = \frac{1}{\sqrt{1+a^2}}$$

(b) Given, $X_L = bR$

$$\therefore \quad Z = \sqrt{R^2 + X_L^2}$$

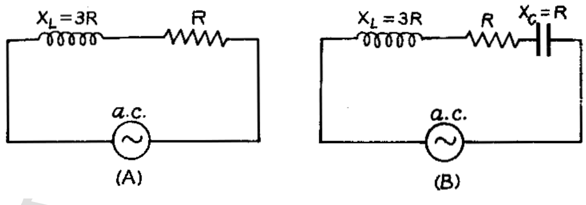
$$= \sqrt{R^2 + b^2R^2} = R\sqrt{1+b^2}$$

$\therefore$ Power factor $\cos\phi = \dfrac{R}{Z}$

$$= \frac{R}{R\sqrt{1+b^2}} = \frac{1}{\sqrt{1+b^2}}$$

$$\therefore \quad \text{Power factor} = \frac{1}{\sqrt{1+b^2}}$$

3) Given below are two electric circuits A and B . Calculate the ratio of power factor of the circuit B to the power factor of circuit A .



SOL: The power factor of the circuit A is

$$\cos\phi_A = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + (3R)^2}} = \frac{1}{\sqrt{10}}.$$

Similarly, the power factor of the circuit B is

$$\cos\phi_B = \frac{R}{\sqrt{R^2 + (X_L^2 - X_C)^2}} = \frac{R}{\sqrt{R^2 + (3R - R)^2}} = \frac{1}{\sqrt{5}}.$$

$\therefore$ ratio of power factor of the circuit B to the circuit A is

$$\frac{\cos\phi_B}{\cos\phi_A} = \frac{1/\sqrt{5}}{1/\sqrt{10}} = \sqrt{2}.$$

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4) **A circuit draws a power of 550 W from a 220 V-50 Hz source. The power factor of the circuit is 0.8. A current in the circuit lags behind the voltage. Show that a capacitor of about $\frac{1}{42\pi} \times 10^{-2}$ F will have to be connected in the circuit to bring its power factor to unity.**

[CBSE 1992 S]

SOL: $I_{rms} = \dfrac{\overline{P}}{V_{rms}\cos\phi} = \dfrac{550}{220 \times 0.8} A = 3.125A$

$$R = \frac{\overline{P}}{I_{rms}^2} = \frac{550}{(3.125)^2} = 56.3\Omega$$

Now using $\tan\phi = \dfrac{\omega L}{R}$, we get $\omega L \approx 42\ \Omega$

Again, $\omega L = \dfrac{1}{\omega C}$ or $C = \dfrac{1}{\omega(\omega L)}$

$$= \frac{1}{100\pi \times 42}\ F = \mathbf{\frac{1}{42\pi} \times 10^{-2}\ F.}$$

5) **A capacitor of capacitance 100 μF and a coil of resistance 50 Ω and inductance 0.5 H are connected in series with a 110 V-50 Hz source. Calculate the rms value of current in the circuit.[CBSE 1998]**

Ans. $X_L = \omega L = 2 \times 3.14 \times 50 \times 0.5\ \Omega = 157\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}}\ \Omega$$

$$= \frac{1000}{31.4}\ \Omega = 31.85\ \Omega$$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{2500 + (157 - 31.85)^2}\ \Omega = 134.77\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{110}{134.77} A = 0.816A$$

6) **A capacitor, resistor of 5Ω and an inductor of 50mH are in series with an ac source marked 100V, 50 Hz. It is found that the voltage is in phase with the current. Calculate the capacitance of the capacitor and the impedance of the .circuit. [CBSE 1999]**

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Ans: Since the voltage is in phase with the current, therefore, it is a case of resonance. The circuit is purely resistive. So, impedance, Z = R = 5 Ω.

$$\text{Again, } f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad C = \frac{1}{4\pi^2 L f^2}$$

$$\text{or} \qquad C = \frac{49}{4 \times 484 \times 50 \times 10^{-3} \times 50 \times 50}\ \text{F}$$

$$= \frac{49}{4 \times 484 \times 125}\ \text{F} = \mathbf{2.02 \times 10^{-4}\ F.}$$

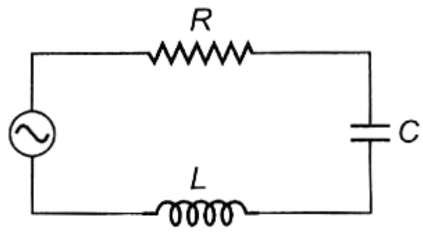
7) The figure shows a series *L-C-R* circuit with *L* = 10.0 H, C = 40 μF, *R* = 60Ω connected to a variable frequency 240 V source, calculate

(a) the angular frequency of the source which drives the circuit at resonance.

(b) the current at the resonating frequency.

(c) the rms potential drop across the inductor at resonance. [Delhi 2012]

SOL:



Given, $L = 10$ H, $C = 40\ \mu$F, $R = 60\ \Omega$,
$V_{\text{rms}} = 240$ V

(a) Resonating angular frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 40 \times 10^{-6}}} = \frac{1}{20 \times 10^{-3}} = 50 rad/s$$

(b) Current at resonating frequency

$$I_{rms} = \frac{V_{rms}}{Z_{\min}} = \frac{V_{rms}}{R} = \frac{240}{60} = 4\text{A} \qquad (\because \text{At resonance } Z_{\min} = R)$$

(c) $\therefore$ Inductive reactance $X_L = \omega L$

At resonance

$$X_L = \omega_0 L = 50 \times 10 = 500\ \Omega$$

Potential drop to across inductor

$$V_{\text{rms}} = I_{\text{rms}} \times X_L$$
$$= 4 \times 500$$
$$V_{\text{rms}} = 2000\ \text{V}$$

8) A 25.0 μ ,F capacitor, a 0.10 H inductor and a 25.0 Ω resistor are connected in series with an a.c. source of emf *E* = 310 sin 314 *t* . Find (i) the frequency of the emf, (ii) the reactance of

the circuit, (iii) the impedance of the circuit, (iv) the current in the circuit and (v) the phase angle. Also find the effective voltages across the capacitor, inductor and resistor.

SOL:

(i) The given emf is $E = 310 \sin 314\, t$.

Comparing it with $E = E_0 \sin \omega t$, we have

$$\omega = 314 \text{ rad s}^{-1}.$$

$$\therefore\ f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3{\cdot}14} = \mathbf{50\ Hz.}$$

(ii) The inductive reactance is

$$X_L = \omega L = 314 \text{ s}^{-1} \times 0{\cdot}10 \text{ H} = 31{\cdot}4\,\Omega.$$

The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \text{ s}^{-1} \times (25{\cdot}0 \times 10^{-6} \text{ F})} = 127{\cdot}4\,\Omega.$$

$\therefore$ net reactance of the circuit is

$$X_C - X_L = 127{\cdot}4\,\Omega - 31{\cdot}4\,\Omega = \mathbf{96}\,\Omega.$$

Since $X_C > X_L$, ***the net reactance is capacitive.***

(iii) The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(25{\cdot}0\,\Omega)^2 + (96\,\Omega)^2} = \mathbf{99{\cdot}2}\,\Omega.$$

(iv) The current in the circuit is

$$i_{rms} = E_{rms}/Z.$$

Here $E_{rms} = E_0/\sqrt{2} = 310/1{\cdot}414 = 219{\cdot}2$ V.

$$\therefore\ i_{rms} = \frac{219{\cdot}2 \text{ V}}{99{\cdot}2\,\Omega} = \mathbf{2{\cdot}21\ A.}$$

(v) The phase angle ϕ is given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{96\,\Omega}{25\,\Omega} = 3{\cdot}84.$$

$$\therefore\ \phi = \tan^{-1}(3{\cdot}84) = \mathbf{75{\cdot}4^\circ}.$$

The effective (rms) voltages across the capacitor, inductor and resistor are

$$V_C = i_{rms} \times X_C = 2{\cdot}21 \text{ A} \times 127{\cdot}4\,\Omega = \mathbf{281{\cdot}6\ V},$$

$$V_L = i_{rms} \times X_L = 2{\cdot}21 \text{ A} \times 31{\cdot}4\,\Omega = \mathbf{69{\cdot}4\ V}$$

and

$$V_R = i_{rms} \times R = 2{\cdot}21 \text{ A} \times 25{\cdot}0\,\Omega = \mathbf{55{\cdot}3\ V}.$$

9) An inductor *L* a capacitor of 20 μF and a resistor of 10Ω are connected in series with an a.c. source of frequency 50 Hz. if the current is in phase with the voltage, calculate the inductance of the inductor.

[ANS: 0.51H]

SOL: $\omega = \omega_0 = 2\pi \times 50 = 100\pi = 314\ rad/s$

In a LCR circuit , the current and the voltage are in phase (ϕ = 0), when

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$$\tan\phi = \frac{\omega_0 L - \frac{1}{\omega_0 C}}{R} = 0$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$L = \frac{1}{\omega_0^{\ 2} C}$$

Here $\omega = 2\pi f = 2 \times 3{\cdot}14 \times 50\ s^{-1} = 314\ s^{-1}$ and $C = 20\ \mu F = 20 \times 10^{-6}\ F$.

$$\therefore\ L = \frac{1}{(314\ s^{-1})^2 \times (20 \times 10^{-6}\ F)} = \mathbf{0{\cdot}51\ H}.$$

10) A capacitor, a 15-Ω resistor and a 80-mH inductor are in series with a 50-Hz a.c. source. Calculate the capacitance if the current is in phase with the voltage?

SOL: In a LCR circuit , the current and the voltage are in phase (ϕ = 0), when

$$\tan\phi = \frac{\omega_0 L - \frac{1}{\omega_0 C}}{R} = 0$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$C = \frac{1}{\omega_0^{\ 2} L}$$

Here $\omega = 2\ \pi f$

$= 2 \times 3{\cdot}14 \times 50\ s^{-1} = 314\ s^{-1}$ and $L = 80\ mH = 80 \times 10^{-3}\ H$.

$$\therefore\ C = \frac{1}{(314\ s^{-1})^2 \times (80 \times 10^{-3}\ H)} = 1{\cdot}27 \times 10^{-4}\ F = \mathbf{127\ \mu F}.$$

11) **A series *L-C* circuit has *L* = 0.405 H and *C* = 25 μF. The resistance *R* is zero. Find** the **natural frequency.**

SOL: The natural frequency of the *L-C* series circuit is given by

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3{\cdot}14\sqrt{0{\cdot}405\ H \times (25 \times 10^{-6}\ F)}} = 50\ Hz.$$

The resonance in the circuit occurs when an a.c. voltage of frequency equal to its natural frequency is applied. Thus, the frequency of resonance is **50 Hz.**

12) In a series *L-C-R* circuit connected to a variable frequency 220-V source; we have ***L* = 4.0 H, C = 100 μF, *R* = 40 Ω. Calculate (i) the resonant frequency of the circuit (ii) *the* impedance of the circuit and the amplitude of current at resonating frequency and (iii) the rim potential drop across *L*.**

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SOL: (i) The resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \times \sqrt{4.0\,\mathrm{H} \times (100 \times 10^{-6}\,\mathrm{F})}} = \mathbf{7.96\ Hz.}$$

(ii) The impedance of the circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R = \mathbf{40\,\Omega},$$

$Z_{min} = R = 40\ \Omega$

Because at resonance , $\omega_0 L = \dfrac{1}{\omega_0 C}$,

The amplitude of current is $I_0 = \dfrac{V_0}{Z_{\min}} = \dfrac{220\sqrt{2}V}{40\Omega} = 5.5\sqrt{2}A$

(iii) The rms potential drop across L is

$$V_L = i_{rms} \times \omega L, \text{ where } \omega = 2\pi f$$

$$= \frac{220\ \mathrm{V}}{40\ \Omega} \times (2 \times 3.14 \times 7.96\,\mathrm{s}^{-1} \times 4.0\,\mathrm{H}) = \mathbf{1100\ V.}$$

13) A series *L-C-R* circuit is made by taking *R* = 100Ω , L = 2/π H and *C* = 100/π μF. This series combination is connected across an a.c. source of 220 V, 50 Hz. Calculate (i) the impedance of the circuit and (ii) the peak value of the current flowing in the circuit. Calculate the power factor of this circuit and compare this value with the one at its resonance frequency.

[All India 2008 C] [ANS: (i)100√2Ω (ii) 2.2A (iii)1]

SOL: The inductive reactance of the circuit is

$$X_L = \omega L = (2\pi \times 50) \times \frac{2}{\pi} = 200\,\Omega.$$

The capacitive reactance of the circuit is

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi \times 50) \times \frac{100}{\pi} \times 10^{-6}} = 100\,\Omega.$$

(i) the impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (200 - 100)^2} = \mathbf{100\sqrt{2}\ \Omega.}$$

(ii) The Peak value of current is $I_0 = \dfrac{V_0}{Z} = \dfrac{\sqrt{2}V_{rms}}{Z} = \dfrac{\sqrt{2} \times 220}{100\sqrt{2}} = 2.2A$

(iii) Power factor of the circuit, $\cos\phi = \dfrac{R}{Z} = \dfrac{100}{100\sqrt{2}} = \dfrac{\mathbf{1}}{\mathbf{\sqrt{2}}}$.

In case of resonance, $Z = R$

$\therefore$ power factor, $\cos\phi = \dfrac{R}{R} = \mathbf{1}$.

14) An inductor 200 mH, capacitor 500μF, resistor 10Ω are connected in series with a 100 V, variable frequency a.c. source. Calculate (i) the frequency at which power factor of the circuit is unity.(ii) Current amplitude at this frequency and (iii) Q-factor.

SOL:

(i) The power factor of the circuit is

$$\cos\phi = \frac{R^2}{\sqrt{R^2 + (X_L - X_C)^2}} = 1$$

$\therefore$ $$R^2 + (X_L - X_C)^2 = R^2$$

or $$X_L = X_C$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3{\cdot}14\sqrt{0{\cdot}2\text{H} \times (5 \times 10^{-4}\text{ F})}} = \frac{100}{6{\cdot}28}$$

f_0 = 15.92 = 16 Hz

(ii) Current amplitude , I_0 = $\frac{V_0}{Z_{\min}} = \frac{V_0}{R} = \frac{100}{10} = 10A$

(iii) Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0{\cdot}2}{5 \times 10^{-4}}}$

$$= \frac{1}{10}\sqrt{\frac{1}{25 \times 10^{-4}}} = \frac{1}{10 \times 5 \times 10^{-2}}$$

$$= \frac{100}{50} = 2.$$

15) **An *LCR* series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/s . When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the *LCR* circuit.**

SOL:

Given : $\tan 60^\circ = \frac{\omega L}{R}$ and $\tan 60^\circ = \frac{1/\omega C}{R}$.

$$\therefore\ \omega L = \frac{1}{\omega C}.$$

The impedance of the circuit is

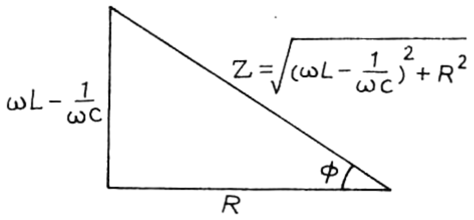
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R.$$

The current in the circuit is

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{200}{100} = 2A$$

Power dispatched in the circuit $\overline{P} = V_{rms} \times I_{rms} \times \cos\phi$

where $\cos\phi$ is the power factor.



Now, $\tan\phi = \dfrac{\omega L - \frac{1}{\omega C}}{R}$.

$\therefore \cos\phi = \dfrac{R}{Z} = \dfrac{R}{R} = 1.$

$$\overline{P} = 200V \times 2A \times 1 = 400W$$

16) A resistor of 12 Ω, a capacitor of reactance 14Ω and a pure inductor of inductance 0.1 H are joined in series and placed across 200 V, 50 Hz ac supply. Calculate (i) the current in the circuit (ii) phase angle between current and voltage. Take π = 3.[CBSE 1993]

SOL:

$R = 12\ \Omega$, $X_C = 14\ \Omega$,

$X_L = \omega L = 2 \times 3 \times 50 \times 0.1\ \Omega = 30\ \Omega$

$X_C = 14\ \Omega$,

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{12^2 + (30-14)^2}\ \Omega = 20\ \Omega.$

(*i*) $I_v = \dfrac{E_v}{Z} = \dfrac{200}{20}\ A = \mathbf{10\ A}$

(*ii*) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{30-14}{12} = 1.3333$

$\phi = \tan^{-1}(1.3333) = \mathbf{53.13^\circ}$

(i) $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{200}{20}A = 10A$

(ii) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{30-14}{12} = 1.3333$

$\phi = \tan^{-1}(1.3333) = 53.13^0$

17) An inductor 200 mH, capacitor 500 μF, resistor 10Ω are connected in series with a 100 V, variable frequency AC source. Calculate the
(a) frequency at which the power factor of the circuit is unity.
(b) current amplitude at this frequency.
(c) Q-factor. [Delhi 2008]

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SOL: (a) When power factor $\cos\phi = 1$; $\phi = 0$ **;**

L-C-R circuit is in resonance.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times10^{-1}\times5\times10^{-4}}} = 100\ rad/\sec$$

$$(b) I_{rms} = \frac{V_{rms}}{Z_{\min}} = \frac{V_{rms}}{R} (at\ resonance\ \mathrm{Z_{min}} = \mathrm{R})$$

$$I_{rms} = \frac{100}{10}$$

$$= 10\ \mathrm{A}$$

Amplitude of maximum current $I_0 = I_{rms}\sqrt{2}$

$$= 10\sqrt{2}\ \mathrm{A}$$

(c) Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{10}\sqrt{\frac{2\times10^{-1}}{5\times10^{-4}}}$$

$$= \frac{1}{10}\sqrt{0.4\times10^{3}}$$

$$= \frac{200}{10} = 20$$

18) In a series LCR circuit, the voltage across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is phase difference between the applied voltage and the current in the circuit ? (CBSE 2007, 05)

Ans: tan $\phi = \frac{V_L - V_C}{V_R}$

19) **In a series *LCR* circuit, the voltage across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is phase difference between the applied voltage and the current in the circuit ?**

ANS. Zero.

20) When a series *LCR* a.c. circuit is brought into resonance, the current has a large value. why?

Ans. The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At resonance ($X_L = X_C$) or ($\omega L = 1/\omega C$), the impedance decreases to R and so the current increases.

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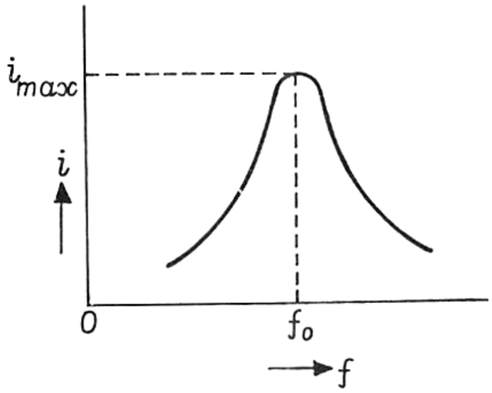
21) **In a series L-C-R circuit, define the quality factor (Q) at resonance. Illustrate its significance by giving one example.**

22) **State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing variation of current with frequency of a.c. source in a series LCR circuit.**

Ans. For resonance, the frequency of the a.c. source showed be equal to the natural frequency of the circuit in the absence of resistance ($f = \dfrac{1}{2\pi\sqrt{LC}}$) The graph is plotted in Fig(a).



23) **An alternating voltage of frequency f is applied across a series L-C-R circuit. Let f_r be the resonance frequency for the circuit. Will be current in the circuit lag, lead or remain in phase with the applied voltage when (i)f > f_r (ii) f < f_r? Explain your answer in each case.**

SOL: (i) Given $f > f_r$

The resonance frequency f_r equals the natural frequency of the circuit (in the absence of R) which is

$\dfrac{1}{2\pi\sqrt{LC}}$,Thus

$$f > \frac{1}{2\pi\sqrt{LC}}$$

$$\omega > \frac{1}{\sqrt{LC}} \qquad [\omega = 2\pi f]$$

$$\omega^2 > \frac{1}{LC}$$

$$\omega L > \frac{1}{\omega C}$$

$$X_L > X_C$$

$$\tan\phi = \frac{(X_L - X_C)}{R} = +ve$$

$$X_L > X_C$$

That is , there is net inductive reactance in the circuit . Hence current will l**ag behind the applied voltage.**

(ii) $f < f_r$

$$f < \frac{1}{2\pi\sqrt{LC}}$$

$$\omega < \frac{1}{\sqrt{LC}} \qquad [\omega = 2\pi f]$$

$$\omega^2 < \frac{1}{LC}$$

$$\omega L < \frac{1}{LC}$$

$$X_L < X_C$$

$$\tan\phi = \frac{(X_L - X_C)}{R} = -ve$$

$$X_C > X_L$$

That is , there is net capacitive reactance in the circuit . Hence current will **lead the applied voltage.**

24) Can the voltage drop across the inductor or the capacitor in a series L-C-R circuit be greater than the applied voltage of the a.c. source ? Justify your answer.

Ans: Yes, in series resonance circuit (ωL = 1 /ωC), the potential differences available across the inductor and across the capacitor may be much more than the applied a.c. voltage.

25) What is the phase difference between the voltage drops across L and C in a series L-C-R circuit connected to an a.c. source ?

Ans. 180°.

26) When are the voltage and current in an L-C-R series a.c. circuit in phase ?

Ans. When the frequency of the applied voltage is $f = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$ = Resonating frequency.

27) Give the phase difference between the applied voltage and current in an L-C-R circuit at resonance.

Ans. Zero.

28) An inductor 200 mH, a capacitor C and a resistor 10 ohm are connected in series with a 100 V, 50 s^{-1} ac source. If the current and voltage are in phase with each other, calculate the capacitance of the capacitor. [CBSE 2006 S]

Sol: As current and voltage are in phase,

$X_L = X_C$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (50)^2 \times 200 \times 10^{-3}}$$

$$= 50.6 \times 10^{-6}\text{ F} = \mathbf{50.6\ \mu F}$$

29) A 100 mH inductor, a 25 µF capacitor and a 15 Ω resistor are connected in series to a 120 V,

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50 Hz ac source. Calculate (i) impedance of the circuit at resonance (ii) current at resonance and (iii) resonant frequency. [CBSE 2000]

SOL: (i) At resonance, the LCR circuit is purely resistive. $Z_{min} = R = 15\ \Omega$

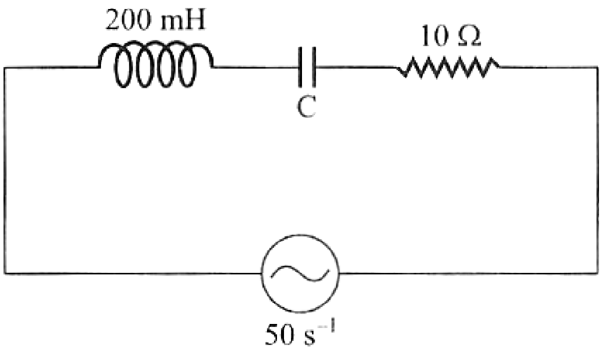
$$(ii)\ \text{Current at resonance} = \frac{120\ \text{V}}{15\ \Omega} = \mathbf{8\ A}$$

$$(iii)\ \text{Resonant frequency, } f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{7}{2 \times 22 \times \sqrt{100 \times 10^{-3} \times 25 \times 10^{-6}}}\ \text{Hz}$$

$$= \frac{31.82}{0.316}\ \text{Hz} = \mathbf{100.7\ Hz}$$

30) In the following circuit, calculate (i) the capacitance 'C' of the capacitor, if the power factor of the circuit is unity, and (ii) also calculate the Q-factor of the circuit. [CBSE 2006 S]



SOL:

$$(i)\ \text{Power factor, } \cos\phi = \frac{R}{Z} = 1 \quad \text{or} \quad Z = R$$

$$\therefore \quad X_C = X_L \quad \text{or} \quad \frac{1}{2\pi f C} = 2\pi f L$$

$$\text{or} \qquad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (50)^2 \times 200 \times 10^{-3}}$$

$$= 5 \times 10^{-5}\,\text{F} = \mathbf{50\ \mu F}$$

$$(ii)\ \text{Q-Factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{5 \times 10^{-5}}} = \mathbf{6.32}$$

31) A series *L-C-R* circuit is connected to a 220 V variable frequency AC supply. If

***L* = 20 mH, C = (800/π^2) μF and *R* = 110 Ω.**

(a) Find the frequency of the source, for which average power absorbed by the circuit is maximum.

(b) Calculate the value of maximum current amplitude. [Delhi 2010C]

SOL:

Given, $V_{rms} = 220$ V, $L = 20$ mH $= 2 \times 10^{-2}$ H,

$R = 110\ \Omega$,

$C = \frac{800}{\pi^2}\ \mu F = \frac{800}{\pi^2} \times 10^{-6}$ F

(a) Average power observed by *L-C-R* series AC circuit is maximum when circuit is in resonance.

∴ Resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2\times10^{-2}\times\frac{800}{\pi^2}\times10^{-6}}} = \frac{1000}{2\times4} = 125 s^{-1}$$

(b) $I_{rms} = \frac{V_{rms}}{Z_{min}} = \frac{V_{rms}}{R} = \frac{220}{110} = 2A$

Maximum current amplitude

$$I_0 = I_{rms}\sqrt{2} = 2\sqrt{2}\ A$$

32) **What is the significance of a *Q* factor in a series *LCR* resonant circuit.**

ANS. It describes quantitatively the sharpness of resonance of the circuit.

33) A capacitor of capacitance 100 µF and a coil of resistance 50Ω and inductance 0.5 H are connected in series with a 110 V - 50 Hz source. Calculate the rms value of the current in the circuit.

ANS: 0.816A

34) A 100-mH inductor, 2041F capacitor and 10-Ω resistor are connected in series to a 100 V-50 Hz a.c. source. Calculate(i) impedance of the circuit at resonance, (ii) current at resonance, and (iii) resonant frequency.

ANS: (i) 10Ω, (ii) 10 A, (iii) 112.6 Hz

35) An inductor of unknown value, a capacitor of 100µF and a resistance of 10 Ω are connected is seres to a 200 V, 50 Hz a.c. source. It is found that the power factor of the circuit is unity. Calculate the inductance of the inductor and the current amplitude.

ANS. 0.101H, 20 A.

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PRACTICE QUESTIONS Transformer (Pg. No. - 94)

EXAMPLES

1) .
2) Draw a labelled diagram of a step-up transformer and explain briefly its working. Deduce the expressions for the secondary voltage and secondary current in terms of the number of turns of primary and secondary windings. How is the power transmission and distribution over long distances done with the use of transformers? [CBSE 2009]
3) Explain with the help of a diagram; the principle, construction and working of a step-up transformer. Give two causes of power loss in it. Why is its core laminated ?
4) Explain with the help of a labelled diagram the underlying principle and working of a step-up transformer. Why cannot such a device be used to step up d.c. voltage ?
5) Describe briefly, with the help of a labelled diagram, working of a step-up transformer. A step up transformer converts low voltage in to high voltage. Does it not violate the principle of conservation of energy ? Explain.
6) (a) With the help of labelled diagram, describe briefly the underlying principle and working of a step-up transformer.

   (b) Describe briefly and two energy losses, giving the reasons for their occurrence in actual transformers. [Foreign 2012]

   (c) A step-up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy? Explain. [Delhi 2011]

7) Explain with the help of a necessary diagram, the working of a step-up transformer and obtain the expression for the transformer equation = $\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$ where the symbols have their usual meaning.

   What are the two main assumptions made to derive the above relations? Mention two important reasons due to which energy losses occur in actual transformer. [All India 2009C]

8) Draw a schematic diagram of a step-up transformer. Explain its working principle.

   Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this relation related to the currents in the two coils?

   How is the transformer used in large scale transmission and distribution of electrical energy over long distance? [All India 2010]

9) The ratio of the number of turns in the primary and the secondary coil of a step-up transformer is 1 : 200. It is connected to A-C mains of 200 V. Calculate the voltage

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developed in the secondary. Determine the current in the secondary, when a current of 2.0 A flows through the primary.

SOL:

If N_p and N_s be the number of turns in the primary and the secondary coils respectively, and V_p and V_s be the primary and the voltages, then we have

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

$$\therefore\ V_s = V_p \times \frac{N_s}{N_p} = 200\ \text{V} \times 200 = \mathbf{40{,}000\ V}.$$

Suppose, the currents through the primary and the secondary are i_p and i_s respectively. Then, assuming that the transformer is 'ideal', we have

$$V_p \times i_p = V_s \times i_s.$$

$$\therefore\ i_s = i_p \times \frac{V_p}{V_s} = 2{\cdot}0\ \text{A} \times \frac{200\ \text{V}}{40{,}000\ \text{V}} = \mathbf{0{\cdot}01\ A}.$$

10) A transformer of 100% efficiency has 200 turns in the primary and 40000 turns in the secondary. It is connected to a 220-V main supply and the secondary feeds to a 100 kΩ resistance. Calculate the output potential difference per turn and the power delivered to the load.

SOL:

$$N_p = 200, N_s = 40000, V_p = 220\ \text{V}, R_s = 100\ \text{k}\Omega = 10^5\ \Omega, V_s = ?\ \text{Now,}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

or

$$V_s = V_p \times \frac{N_s}{N_p} = 220\ \text{V} \times \frac{40000}{200} = 44000\ \text{V}.$$

The output potential difference per turn is

$$\frac{V_s}{N_s} = \frac{44000\ \text{V}}{40000} = \mathbf{1{\cdot}1\ V.}$$

The power delivered by the 'ideal' (100% efficiency) transformer to the load is

$$V_s \times i_s = V_s \times \frac{V_s}{R_s} = \frac{V_s^{\ 2}}{R_s} = \frac{(44000\ \text{V})^2}{10^5\ \Omega} = 19360\ \text{W} = \mathbf{19{\cdot}36\ kW}.$$

11) A step-down transformer drops the main supply voltage of 220 V to 10 V. The primary draws a **current of 5 A and the current induced in the secondary is 100 A. Calculate the efficiency of the transformer.**

SOL:

$V_p = 220$ V, $V_s = 10$ V, $i_p = 5$ A, $i_s = 100$ A. Now,

$$\text{power input} = V_p \times i_p = 220 \times 5 = 1100 \text{ W}$$

$$\text{power output} = V_s \times i_s = 10 \times 100 = 1000 \text{ W}.$$

$$\text{efficiency} = \frac{\text{power output}}{\text{power input}} = \frac{1000 \text{ W}}{1100 \text{ W}} = 0{\cdot}91 = \mathbf{91\%}.$$

12) Calculate the current drawn by the primary of a transformer which steps down 200 V to 20 V to operate a device of resistance 20 Ω. Assume the efficiency of transformer to be 80%.

[ANS: 0.125A]

SOL:

$$V_p = 200 \text{ V}, V_s = 20 \text{ V}, i_s = \frac{V_s}{R} = \frac{20 \text{ V}}{20\,\Omega} = 1 \text{ A}, i_p = ?$$

The efficiency of the transformer is

$$\eta = \frac{\text{power output}}{\text{power input}} = \frac{V_s \times i_s}{V_p \times i_p}.$$

Here $\eta = 80\% = 0{\cdot}8$

$$\therefore \quad 0{\cdot}8 = \frac{20 \text{ V} \times 1 \text{ A}}{200 \text{ V} \times i_p}$$

or

$$i_p = \frac{20 \text{ V} \times 1 \text{ A}}{200 \text{ V} \times 0{\cdot}8} = \mathbf{0{\cdot}125 \text{ A}}.$$

13) **An alternating emf of 120 V is applied to the primary of a step-up transformer. The current in the primary is 1.85 A while that induced in the secondary is 150 mA. Find the voltage across the secondary, assuming 95 % efficiency.**

SOL:

$$V_p = 120 \text{ V}, i_p = 1{\cdot}85 \text{ A}, i_s = 150 \text{ mA} = 0{\cdot}15 \text{ A}, \eta = 95\% = 0{\cdot}95, V_s = ?$$

Now,

$$\eta = \frac{\text{power output}}{\text{power input}} = \frac{V_s \times i_s}{V_p \times i_p}.$$

$$\therefore \; V_s = \eta \times \frac{V_p \times i_p}{i_s} = 0{\cdot}95 \times \frac{120 \text{ V} \times 1{\cdot}85 \text{ A}}{0{\cdot}15 \text{ A}} = \mathbf{1406 \text{ V}}.$$

14) 11 kW of electric power can be transmitted to a distant station at (i) 220 V or (ii) 22000 V. Which of the two modes of transmission should be preferred and why ? Support your answer with possible calculations. [CBSE 1998]

SOL:

$$(i) \text{ Current, } I_1 = \frac{P}{V_1} = \frac{11000 \text{ W}}{220 \text{ V}} = 50 \text{ A}$$

$$\text{Power loss} = I_1^2 R = 50 \times 50 \times R \text{ watt} = 2500 \text{ R watt}$$

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(*ii*) Current, $I_2 = \frac{P}{V_2} = \frac{11000}{22000}$ A = 0.5 A

Power loss $= I_2^2R = 0.5 \times 0.5 \times R$ watt $= 0.25$ R watt

The power loss is clearly $\frac{2500}{0.25}$ or 10^4 times more in the first case.

So, second mode of transmission should be preferred.

15) A step-down transformer operated on a 2.5 kV line. It supplies a load with 20 A. The ratio of the primary winding to the secondary is 10 : 1. If the transformer is 90% efficient, calculate

a) the power output,

b) the voltage, and

c) the current in the secondary. [Foreign 2010]

SOL:

Input voltage $V_p = 2.5 \times 10^3$ V

Input current $I_p = 20$ A

Also, $\frac{N_p}{N_s} = \frac{10}{1}$

$\Rightarrow \frac{N_s}{N_p} = \frac{1}{10}$

Percentage efficiency = $\frac{\text{Output power}}{\text{Input power}} \times 100$

$$\frac{90}{100} = \frac{\text{Output power}}{V_p I_p}$$

(a) Output power $= \frac{90}{100} \times (V_p I_p)$

$= \frac{90}{100} \times (2.5 \times 10^3) \times (20\text{ A})$

$= 4.5 \times 10^4$ W

(b) $\because \frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\Rightarrow V_s = \frac{N_s}{N_p} \times V_p$

Voltage $V_s = \frac{1}{10} \times 2.5 \times 10^3$ V= 250 V

(c) $V_s I_s = 4.5 \times 10^4$ W

Current $I_s = \frac{4.5 \times 10^4}{V_s} = \frac{4.5 \times 10^4}{250}$

$= 180$ A