MODERN PHYSICS

PRACTICE QUESTIONS (PHOTO ELECTRIC EFFECT) Pg No 18

1) **Define 'intensity' of radiation in photon picture of light. [Comptt. Delhi 2012]**

SOL: It is the number of photo electrons emitted per second.

2) **Define the term 'stopping potential' in relation to photoelectric effect. [All India 2011]**

SOL: In experimental set up of photoelectric effect, a value of negative potential of anode at which photoelectric current in the circuit reduces to zero is called stopping potential or cut-off potential for the given frequency of the incident radiation.

3) **Write Einstein's photoelectric equation. Explain the terms**
   a) **Threshold frequency and**
   b) **Stopping potential. [Delhi 2008C]**

SOL: Einstein's photoelectric equation $E = W_0 + K_{max}$

$$h\ \nu = h\ \nu_o + \frac{1}{2} m v_{max}^2$$

a) Threshold frequency (ν_0) : The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency.

4) **State the dependence of work function on the kinetic energy of the electrons emitted in a photocell. If the intensity of incident radiation is doubled, what changes occur in the stopping potential, kinetic energy of the photoelectrons and the photoelectric current.**

SOL: The maximum Kinetic energy of photoelectrons is related to the work function W_o through Einstein's photoelectric equation

$$K_{max} = \frac{1}{2} m v^2 = h\ \nu - W_o$$

Thus , greater the work function of the surface, lesser is the kinetic energy of the photoelectrons.Vo and K_{max} are independent of incident intensity, but photoelectric current increases with increasing intensity.

5) **On what factor does the magnitude of photoelectric current, the velocity of ejected electrons depend during photoelectric emission from a metal ?**

SOL: The photoelectric current depends upon the intensity, while the velocity of photoelectrons depends upon the frequency of the incident light.

6) **On what factors does the maximum kinetic energy of photoelectrons in a photocell depend ?**

SOL: It depends on the frequency of incident radiation and the work function of the photosensitive material used in the photocell.

7) **What is 'threshold wavelength' in photoelectric effect ?**

SOL: If the wavelength of light incident on a metal exceeds a certain value, no photoelectrons are emitted from the metal. The maximum wavelength of light capable of ejecting photoelectrons from a metal is called the 'threshold wavelength' for that metal.

8) What is meant by 'threshold frequency' in photoelectric effect? Does it depend upon the intensity of incident light ?

SOL: The minimum frequency of incident light capable of ejecting photoelectrons from a metal is called 'threshold frequency' for that metal. It does not depend upon the intensity of light.

9) What is the relation between the work function W and the threshold frequency ν_0 of a metal?

SOL: $W = h\nu_0$.

10) Which of the following radiations will be most effective for electron-emission from the surface of zinc ? Microwave, infra-red, ultra violet.

Ans: Ultra-violet, because its photons, having maximum energy, can eject photoelectrons from metals of high work function.

11) The work function of lithium is 2.3 eV. What does it mean?

SOL: For the emission of photoelectrons from the lithium metal, the minimum energy of light-photon incident on the metal surface should be 2.3 eV.

12) For a photoemissive surface , threshold wavelength is λ_0. Does photoemission occur if the wavelength of incident radiation is (i) more than λ_0 (ii) less than λ_0?

SOL: (i) No.(ii) Yes

13) What is the relation between the work function W and threshold wavelength λ_0 of the metal?

SOL: $W = hc/\lambda_0$

14) Two metals A and B have work functions 2eV and 4eV respectively. Which one has a lower threshold wavelength for photoelectric emission?

SOL: Metal B

15) In photoelectric effect, why should the photoelectric current increase as the intensity of monochromatic radiation incident on a photosensitive surface is increased ? Explain. [Foreign 2014]

SOL: Increase in intensity of the incident radiation corresponds to an increase in the number of incidents photons, resulting in the an increase in the number of photo electrons emitted.

16) Show the variation of photocurrent with collector plate potential for different frequencies but same intensity of incident radiation. [Foreign 2011]

17) In a plot of photoelectric current *versus* anode potential, how does

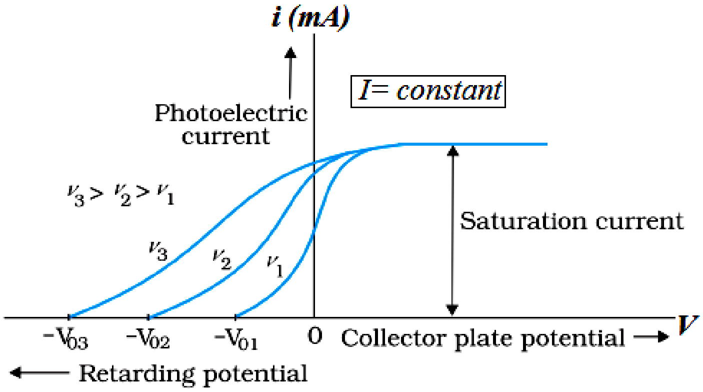
a) The saturation current vary with anode potential for incident radiations of different frequencies but same intensity?

b) The stopping potential vary for incident radiations of different intensities but same frequency?

c) Photoelectric current vary for different intensities but same frequency of incident radiations?

Justify your answer in each case. [Delhi 2007]

SOL: The variation of photocurrent with collector plate potential for different frequencies is shown below.



(a) The saturation current remain same for incident radiation of same intensities and different frequencies with anode potential as number of photoelectrons ejected from metal surface remain same with the constancy of intensity of incident light beam.

(b) Stopping potential does not vary for incident radiations of different intensities but same frequency as energy of each incident photon remain same which in turn ejects photoelectron of same KE_{max} for given metal and hence stopping potential remains same.

(c) With the increase of intensity of incident light beam, number of photons incident per unit time per unit area increase whereas energy of each photon remain same due to constancy of frequency. This lead to eject more photoelectrons from surface and current grows with the corresponding increase of anode potential.

Also, the saturation photocurrent increases with the increase of intensity whereas cut-off potential remains same.

18) Draw a plot showing the variation of photoelectric current with collector plate potential for two different frequencies, $\nu_2 > \nu_1$ of incident radiation having the same intensity. In which case will the stopping potential be higher? Justify your answer. [All India 2011]

SOL: Stopping potential will be higher corresponding to frequency ν_2.

By Einstein's photoelectric equation

$$eV_o = h\nu - h\nu_o$$

$$\Rightarrow V_o = \frac{h}{e}\nu - \frac{h}{e}\nu_o$$

$$V_o = \frac{h}{e}\nu - \frac{W_o}{e}$$

where, V_o = cut-off potential,

h = Planck's constant

e = electronic charge of photosensitive

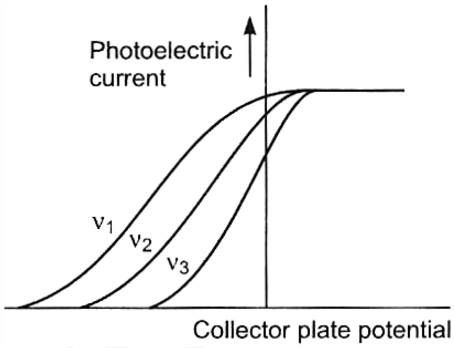
W_o = work function of material

It is clear that for higher frequency ν, cut-off potential is higher.

19) The given graph shows variation of photoelectric current with collector plate potential for different frequencies of incident radiations.

(a) Which physical parameter is kept constant for the three curves?

(b) Which frequencies (ν_1, ν_2 or ν_3) is the highest? [Foreign 2009]

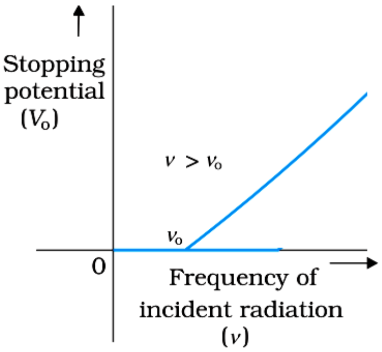


SOL: (a) According to laws of photoelectric emission, the photoelectric current depends on the intensify of incident light. The constant saturation value of photoelectric current reveals that intensity of incident light is constant.

(b) Frequency ν_1 is the highest among ν_1, ν_2 and ν_3 because higher the cut-off potential, higher the frequency of incident light.

20) Show graphically how the stopping potential for a given photosensitive surface varies with frequency of incident radiations? [Foreign 2007]

SOL:

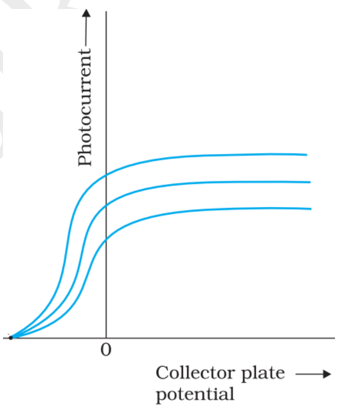


21) What is threshold frequency in photoelectric emission ? The frequency (ν) of incident radiation is greater than the threshold frequency (ν_0) in a photocell. How will the stopping potential V_0 vary, if frequency (ν) is increased, keeping other factors constant.

SOL: The threshold frequency ν_o for a metal is the minimum frequency of incident light which can eject photoelectrons from that metal. Below ν_o, there is no photoelectric emission.

When $\nu > \nu_o$, the stopping potential varies linearly with frequency ν of the incident light.

22) **In an experiment on photoelectric effect, the adjoining graphs are obtained between the photoelectric current (i) and the anode potential (V) . Name the characteristic of the incident radiation that was kept constant in this experiment.**



Ans: Frequency.

23) A source of light is placed at a distance of 50 cm from a photocell and the cut off potential is found to be V_o . If the distance between the light source and the photocell is made 25 cm, what will be the new cut off potential ? Justify your answer.

SOL: The new cut-off potential is Vo , same as before. The cut-off potential depends upon the frequency of incident light, and not on intensity of light. On bringing the light source nearer the photocell, the frequency of light remains unchanged, only intensity changes.

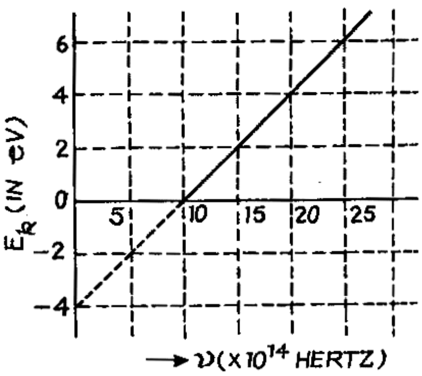
24) Red light, however bright it is, can not produce the emission of electrons from a clean zinc surface. But even weak ultraviolet radiation can do so. Why?

SOL: The frequency of red light is lower than the threshold frequency for zinc surface. On the other hand, the frequency of ultraviolet radiation is much higher than the threshold frequency for zinc.

25) When a monochromatic yellow coloured light beam is incident on a photosensitive surface, photoelectrons are not ejected, while the same surface gives photoelectrons when exposed to green coloured monochromatic beam. What will happen if the same surface is exposed to (i) violet, (ii) red coloured monochromatic beam of light ? Justify your answer.

SOL: (i) Photoelectrons will be ejected, because the frequency of violet beam is still higher than that of green beam which ejects photoelectrons.
(ii) Photoelectrons will not be ejected, because the frequency of red beam is still lower than that of yellow beam which does not elect Photoelectrons.

26) In an experiment of photoelectric effect, the graph between the maximum kinetic energy E_k of the emitted photoelectrons and the frequency υ of the incident light is a straight line shown in the adjoining figure. Calculate : (a) threshold frequency, (b) work function W of cathode-metal in eV, (c) Planck's constant *h* and (d) maximum kinetic energy of the electrons emitted by light of frequency $\nu = 20 \times 10^{14}$ second^{-1} .



Ans. (a) 10×10^{14}, (b) 4 eV, (c) 6.4×10^{-34} J s, (d) 4 eV

27) Two metals X and Y have work functions 2 eV and 5 eV respectively. Which metal will emit electrons, when irradiated with light of wavelength 400 nm and why ?

SOL: Metal X , because a photon of wavelength 400 nm carries energy about 3 eV (E = h c/λ).

28) The work function of a metal is 3.2 eV. Light of wavelength 300 nm is falling on the surface of this metal. Find the kinetic energy of the fastest photoelectrons emitted. [ANS: 0.925 eV]

29) The stopping potential in an experiment on photoelectric effect is 1.5 V. What is the maximum kinetic energy of the photoelectrons emitted? [All India 2009]

SOL: $K_{\max} = eV_o = e \times 1.5\ Volt = 1.5eV$.

30) The maximum kinetic energy of a photoelectron is 3 eV. What is its stopping potential? [All India 2009]

SOL: Maximum kinetic energy of photoelectron = 3 eV

$K_{max} = eV_o$,   where, V_o = stopping potential

$3\ eV = e\ V_o$

∴ Stopping potential $V_o = 3$ V

31) The stopping potential in an experiment on photoelectric effect is 2V. What is the maximum kinetic energy of the photoelectrons emitted? [All India 2009]

SOL: Given stopping potential in an experiment on photoelectric effect = 2 V

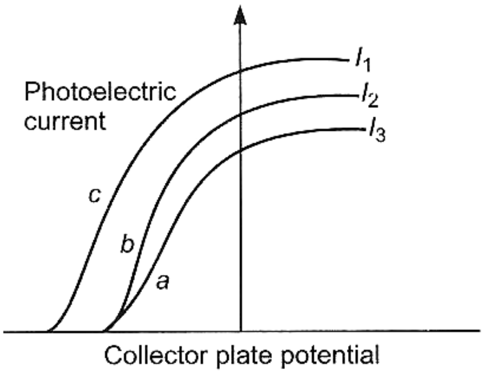
Maximum kinetic energy $K_{max} = eV_o = e(2\ V) = 2$ eV.

32) **If the maximum kinetic energy of electrons emitted by a photo-cell is 5 eV, what is its stopping potential ?**

Ans: 5 V.

33) The figure shows a plot of three curves a, *b*, c showing the variation of photocurrent vs collector plate potential for three different intensities I_1, I_2 and I_3 having frequencies ν_1, ν_2 and ν_3 respectively incident on a photosensitive surface.

Point out the two curves for which the incident radiations have same frequency but different intensities.   [Delhi 2009]



SOL: Curves *a* and *b* have got same cut-off potential, so for these two curves frequencies will be same.

34) Ultraviolet radiations of different frequencies ν_1 and ν_2 are incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$) respectively. The kinetic energy of the emitted electrons is same in both the cases. Which one of the two radiations will be higher frequency? [All India 2007]

SOL: KE of photo electrons $K = h\nu - W$

Here, kinetic energies of electrons in both the cases is same.

$(K)_1 = (K)_2$

$(h\nu_1 - W_1) = (h\nu_2 - W_2)$

Here, $W_1 > W_2$

$\nu_1 > \nu_2$

35) What is meant by work function of a metal ? Ultraviolet light is incident on two photosensitive materials having work functions W_1 and W_2 , (W_1, > W_2). In which case will the kinetic energy of the emitted electrons be greater ? Why ?

SOL: The work function of a metal is the minimum energy required to pull out an electron from the surface of the metal, without imparting any energy to the electron.
By Einstein's photoelectric equation, we have

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = h\nu - W_o \quad .$$

Where ν is the frequency of the incident radiation. Clearly, if work function W increases, the kinetic energy K_{max} of the librated photoelectrons decreases. Here

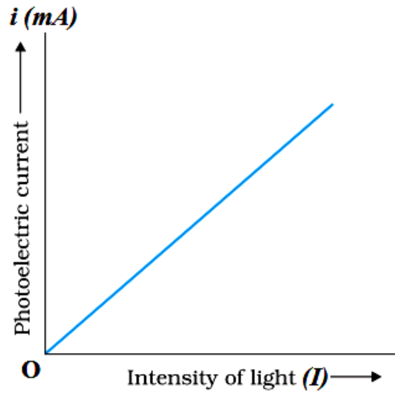
$W_2 < W_1 \qquad \therefore K_{\max(2)} > K_{\max(2)}$

That is , the kinetic energy of ht eemitted electrons will be greater in the second case.

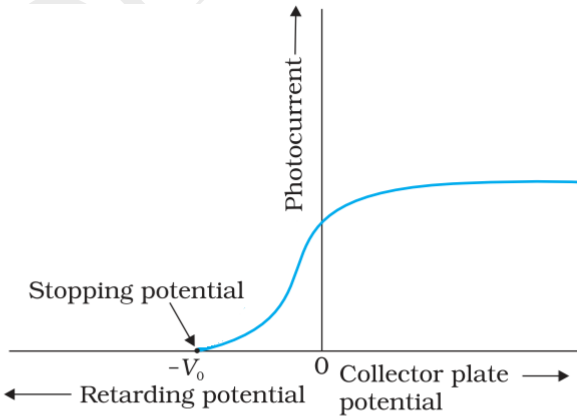
36) Yellow light incident on the cathode of a photoelectric cell is replaced by blue light and the intensity of light is also increased. The current from the cell increases and the potential difference required to stop the current completely also increases. State which effect is produced due to which change.

SOL: The current increases due to increase in intensity, while the stopping potential difference increases due to replacement of yellow light by blue light (that is, due to increase in frequency of the incident light).

37) Explain by drawing graphs, the variation of photoelectric current in a photo-tube with (i) voltage across the tube and (ii) intensity of incident radiation.



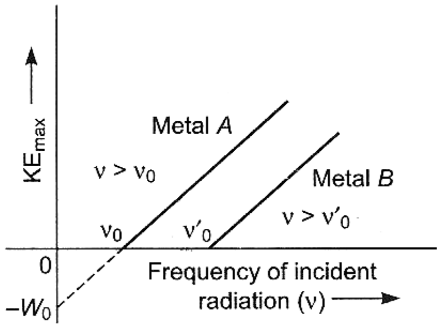
SOL: (*i*) At all positive voltages of anode, the photoelectric current remains maximum because *all* photoelectrons emitted from the cathode reach the anode. As the anode is made more and more negative relative to cathode, an increasing number of emitted electrons fail to reach the anode and so the current falls. At a certain (negative) cut-off voltage V_0 , the current stops.



(ii) With increasing intensity of incident radiation, the rate of electron emission, and hence the current, increases linearly. Hence the graph between current and intensity is a straight line through the origin.

38) Show graphically how the maximum kinetic energy of electrons emitted from a photosensitive surface varies with the frequency of incident radiations? [Foreign 2007]

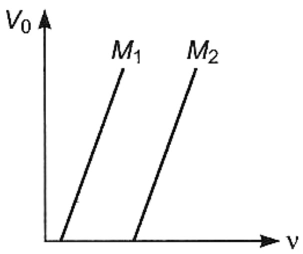
SOL: The variation of maximum kinetic energy with frequencies of incident radiations is shown below.



39) Figure shows variation of stopping potential (V_0) with the frequency (ν) for two photosensitive materials M_1 and M_2.

a) Why is the slope same for both lines?

b) For which material will the emitted electrons have greater kinetic energy for the incident radiation of the same frequency? Justify your answer.



SOL: By the Einstein's photoelectric equation

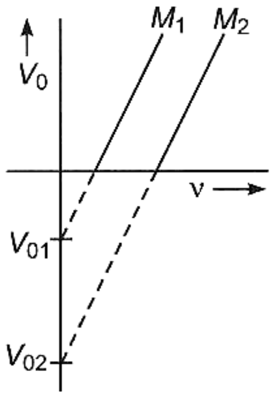
$$\frac{1}{2}m\text{v}_{\max}^2 = h\ \nu - h\ \nu_o \ldots\text{(i)}$$

Since the stopping potential potential just stops the maximum-energy photoelectrons, we have

$$\frac{1}{2}m\text{v}_{\max}^2 = eV_o \ldots\text{(ii)}$$

From (i) and (ii) $eV_0 = h\ \nu - \text{h}\ \nu_o$

$$V_0 = \left(\frac{h}{e}\right)\ \nu - \left(\frac{h}{e}\right)\nu_o \ \ldots\text{..(iii)}$$



This is the equation of the V_0 versus ν graph. It represents a straight line since h and e are constants and ν_o is the constant for a given surface.

Comparing equation (iii) with the equation of line y = mx + c.

Slope m = $\frac{h}{e}$, where h is Planck's constant and e is charge of electron. Moreover the ratio $\frac{h}{e}$ is universal constant. Hence the slope of V_o versus ν graph is same for all metals.

(b) The intercept for material M_1 is more (less negative) than the material M_2 and at the same time $V_o \propto K_{\max}$ therefore the kinetic energy of the photo electrons for material M_1 is more , than M_2.

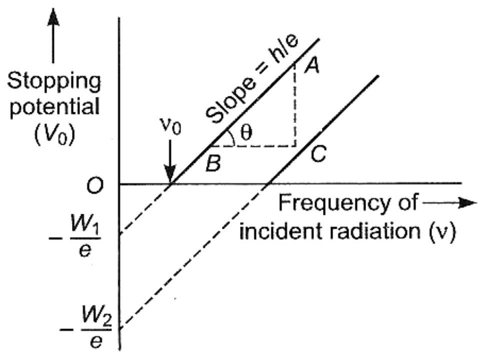
40) Plot a graph showing the variation of stopping potential with the frequency of incident radiation for two different photosensitizes materials having work functions W_1 and W_2 ($W_1 > W_2$). On what factors does the

a) slope and

b) intercept of the lines depend? [Delhi 2010]

Write *two* important conclusions that can be drawn from the study of these plots. [Comptt Outside Delhi 2012]

SOL:



a) The slope of stopping potential *versus* frequency of incident radiation gives the ratio of Planck's constant *(h)* and electronic charge (e).

b) Intercept on the frequency axis gives the value of threshold frequency ν_o.

Intercept on the potential axis = $-\frac{h\nu_o}{e} = -\frac{W_o}{e}$.

<u>Conclusions:</u>

$W_1 = \nu_{01}$ $\qquad W_2 = h\nu_{02}$

Given : $W_1 > W_2$ $\qquad \nu_{01} > \nu_{02}$

(i) Threshold frequency of material having work function W_1 is more than that of material of work function W_2.

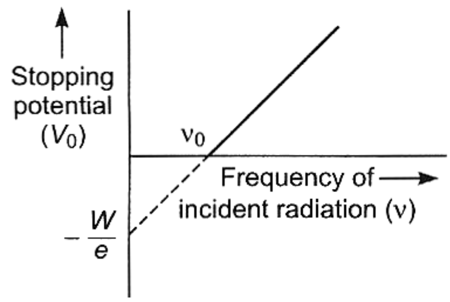
(ii) The slopes of the straight line graphs, in both the cases, have the same value.

(iii) For the same frequency of incident radiation (> ν_{01}), the maximum kinetic energy of the electrons, emitted from the material of work function W_1 is < that electrons emitted from material of work function W_2.

41) **Write Einstein's photoelectric equation in terms of the stopping potential and the threshold frequency for a given photosensitive material. Draw a plot showing the variation of stopping potential *vs* the frequency of incident radiation. [All India 2008]**

42) **Plot a graph showing variation of stopping potential (V_o) with the frequency (ν) of the incident radiation for a given photosensitive material. Hence, state the significance of the threshold frequency in photoelectric emission. Using the principle of energy conservation, write the equation relating the energy of incident photon, threshold frequency and the maximum kinetic energy of the emitted photoelectrons.**

[Delhi 2009C]



SOL: The graph showing the variation of stopping potential with the frequency is shown below.
W = Work-function of photosensitive material
e = Electronic charge.
From Einstein's photo electric equation

$(K)_{max} = h\nu - h\nu_o = h(\nu - \nu_o)$

where ν = frequency of incident radiation,

ν_o = threshold frequency.

From the equation, is $\nu < \nu_o$ then $(K)_{max}$ is negative, which is not possible. For a given material, there exists a certain minimum frequency of the of the incident radiation below which no emission of photoelectrons take place, this frequency is called **threshold frequency** *i.e.* photoelectric effect will take place only when $\nu > \nu_o$

43) **Write two characteristic features observed is photoelectric effect which supports the photon pictures of electromagnetic radiation.**

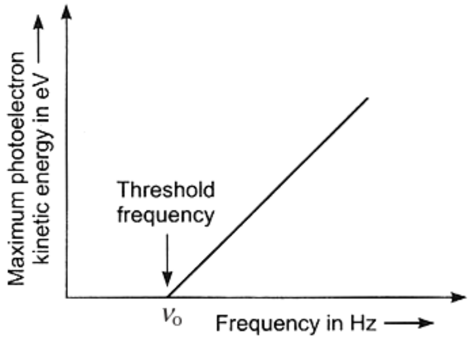
Draw a graph between the frequency of incident radiation (ν) and the maximum kinetic energy of the electrons emitted from the surface of a photosensitive material state clearly how this graph can be used to determine (a) Planck's constant and (b) work function of the material? [Foreign 2012]

SOL: The two characteristics features observed in photoelectric effect which support the photon pictures of electromagnetic radiation one **.**

(i) All photons of light of a particular frequency ν, or wavelength λ, have the same energy $E\left(= h\nu = \frac{hc}{\lambda}\right)$ and momentum

$p\left(=\frac{h}{\lambda}\right)$ whatever the intensity of radiation may be. The increase in intensity of the radiation implies an increase in the number of photons

(ii) Photons are electrically neutral and are not deflected by electric and magnetic fields.



(a) Planck constant is given by slope of the curve *i.e.*,Slope of graph = $\frac{h}{e}$.

(b) Work function is the minimum energy required by the electron to escape out of the metal surface thus, $W_o = h\nu_o$

Here, ν_o is the threshold frequency.

44) Define the terms 'cut-off voltage' and 'threshold frequency' in relation to the phenomenon of photoelectric effect. Using Einstein's photoelectric equation show how the cut-off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph. [All India 2012]

SOL: Cut-off voltage The minimum negative voltage (V_0) applied an anode plate for which photocurrent in the circuit reduces to zero.

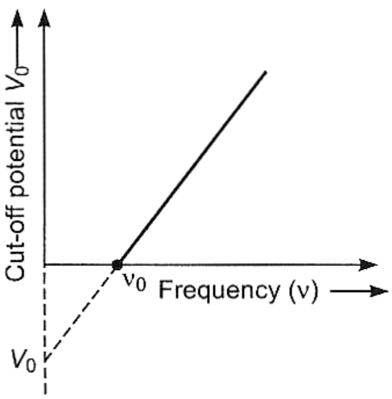
Threshold frequency The minimum frequency of incident radiation which is required for photoelectric effect or the ejection of photoelectrons from metal surface.

Einstein's equation,

$h\nu = h\nu_o + K_{rnax} = h\nu_o + eV_o$

$V_0 = \frac{h}{e}(\nu - \nu_o)$

The variation of cut-off potential with frequency of incident radiation is shown below.



From this graph, we can calculate the wave of threshold frequency (point of intersection of frequency axis) and stopping potential (point of intersection on potential axis).

45) (a) Ultraviolet light of wavelength 2271 $\overset{o}{A}$ from a 100 W mercury source is incident on a photocell made of molybdenum metal. If the stopping potential is 1.3 V, estimate the work function of the metal.

(b) How would the photocell respond to high intensity? (10^5 W/m^2) red light of wavelength 6328 $\overset{o}{A}$ produced by a He-Ne laser? [Delhi 2011C]

SOL: (a) Here $h = 6.63 \times 10^{-34}\, Js$, $\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{2271\times10^{-10}} = 1.32\times10^{15}\,Hz$

V_o = 1.3 V

Einstein's photoelectric equation is

$K_{max} = h\nu - W_0$

$K_{max} = eV_0$

$eV_0 = h\nu - W_0$

$W_0 = h\nu - eV_0$

Work function = $h\nu - eV_o = (6.63\times10^{-34})\times(1.32\times10^{15}) - (1.6\times10^{-19})\times1.3$

$$= 8.76\times10^{-19} - 2\times10^{-19} = 6.76\times10^{-19}\,J = \frac{6.76\times10^{-19}}{1.6\times10^{-19}} = 4.22eV$$

(b) $\lambda = 6328\times10^{-10}$ m

$$h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{6328\times10^{-10}} = 1.96\ eV$$

$h\nu < W_0$

$\therefore K_{max} < 0$

Which is not possible hence Photoelectric effect does not take place.

46) Explain briefly the reasons why wave theory of light is not able to explain the observed features in photoelectric effect? [Foreign 2010]

SOL: The wave theory of light is not able to explain the observed features of photoelectric current because of following reasons.

(a) The greater energy incident per unit time per unit area with the increase of intensity which should facilitate liberation of photoelectron of greater kinetic energy which is in contradiction of observed feature of photoelectric effect.

(b) Wave theory states that energy carried by wave is independent of frequency of light wave and hence wave of high intensity and low frequency (less than threshold frequency) should stimulate photoelectric emission but practically it does not happen.

47) (a)Why photoelectric effect can not be explained on the basis of wave nature of light ? Give reasons.

(b) Write the basic features of photon picture of electromagnetic radiation on which Einstein' s photoelectric equation is based.

SOL: (a)There are threshold of energy (frequency) to cause photo emission. If it was on basis of wave theory , there would have been threshold of intensity.

(b) $E = W + KE_{max}$

Photons are absorbed by electrons.

48) Write three characteristic features in photoelectric effect which cannot be explained on the basis of wave theory of light, but can be explained only using Einstein's equation. [Delhi 2016]

SOL: The three characterist ic features which can't be explained by wave theory are:

i. Kinetic energy of emitted electrons are found to be independent of intensity of incident light.

ii. Below a certain frequency (threshold) there is no photo-emission.

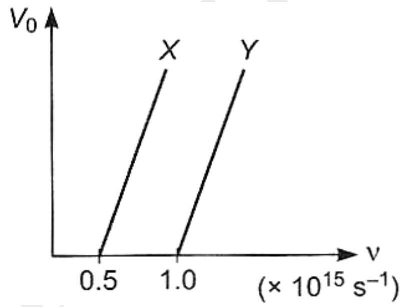
iii. Spontaneous emission of photo-electrons.

49) The following graph shows the variation of stopping potential V_o with the frequency ν of the incident radiation for two photosensitive metals X and Y

a) Which of the metals has larger threshold wavelength? Give reason.

b) Explain, giving reason, which metal gives out electrons, having larger kinetic energy. For the same wavelength of the incident radiation.

c) If the distance between the Light source and metal X is halved, how will the kinetic energy of electrons emitted from it change? Give reason. [All India 2008]



SOL: (a) From graph, threshold frequency for material X $\nu_x = 0.5 \times 10^{15}\ s^{-1}$.
Threshold frequency for material Y, $\nu_Y = 1 \times 10^{15}\ s^{-1}$
It is clear that $\nu_Y > \nu_X$

Threshold wavelength $\lambda = \frac{c}{\nu}$ *i.e.* $\lambda \propto \frac{1}{\nu}$

$\lambda_Y > \lambda_X$

(b) $K_{max} = h\nu - h\nu_o$; ν_o is higher for metal Y for given

wavelength and hence frequency ν .
K_{max} for metal X is greater than the metal Y.
(c) No effect as kinetic energy depends only on frequency of incident electron beam.

50) Radiations of frequency 10^{15} Hz are incident on two photosensitive surfaces A and *B*.

Following observations are recorded.

Surface A No photo-emission takes place.

Surface *B* Photo-emission takes place but photoelectrons have zero energy.

Explain the above observations on the basis of Einstein's photo-electric equation. How will the observation with surface *B* change when the wavelength of incident radiations is decreased? [Delhi 2007]

SOL: Einstein's photoelectric equation states that
$K_{max} = h(\nu - \nu_o)$ ...(i)
where, ν = frequency of incident light
ν_o = threshold frequency.
For surface A: No photo emission takes places from surface A, it means $\nu < \nu_o$ i.e., incident frequency is less than threshold frequency because K_{max} can never be negative.
For surface B: $K_{max} = 0$
$h(\nu - \nu_o) = 0$
$\nu = \nu_o$
i.e., frequency of incident light is equal to the threshold frequency of the photosensitive material.
With the decrease of wavelength of light of incident beam, frequency increases and hence energy of incident photon increases that in turn increases the maximum K_{max} of photoelectron by given photosensitive material (of fixed work function W_o) as

$K_{max} = h\nu - W_o$.

51) Sketch a graph between frequency of incident radiations and stopping potential for a given photosensitive material. What information can be obtained from the value of the intercept on the potential axis? A source of light of frequency greater than the threshold frequency is placed at a distance of 1 m from the cathode of a photo-cell. The stopping potential is found to be V. If the distance of the light source from the cathode is reduced, explain giving reasons, what change will you observe in the

a) Photoelectric current,

b) Stopping potential. [All India 2006]

SOL: The value of intercept on potential axis gives the ratio of work function (W_o) of photosensitive material and electronic charge.
When the distance between light source from the cathode is reduced then intensity of light incident on cathode increases that will facilitate.
(a) the more number of ejection of photoelectrons and hence, saturation value of photocurrent increases.

(b) no change in cut-off potential and intensity increase does not effect the frequency of incident beam and energy of each photon and hence maximum kinetic energy K_{max} of photoelectron remains same and which result in the form of constancy of stopping potential.

52) Light of wavelength 2000 Å, falls on a metal surface of work function 4.2 eV. What is the kinetic energy (in eV) of (i) the fastest and (ii) the slowest photoelectrons emitted from the surface? [CBSE Sample Paper]

SOL: Wavelength of incident light (λ) = 2000 Å **,** Work function (W_0) = 4.2 eV

From Einstein's photoelectric equation

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = h\nu - W_o$$

$$K_{\max} = \frac{hc}{\lambda} - W_o$$

Maximum kinetic energy for fastest photo electron,

$$= \left[\frac{6.63\times10^{-34}\times3\times10^{8}}{2000\times10^{-10}\times1.6\times10^{-19}} - 4.2\right]\text{eV}$$

=1.99 eV

The K_{max} of the slowest emitted photo electron is zero.

53) Which two main observations in photoelectricity led Einstein to suggest the photon theory for the interaction of light with the free electrons in a metal ? Obtain an expression for the threshold frequency for photoelectric emission in terms of the work function of the metal. [CBSE Sample Paper]

SOL: Two main observations are given below.

(i) The maximum kinetic energy of the emitted photoelectrons is independent of the intensity of incident light but depends only upon the frequency of light.
(ii) For each photoemitter, there exists a threshold frequency below which no emission takes place.

According to Einstein, the intensity of light depends on the number of photons.

When radiation of frequency v falls on a metal surface, the energy of each incident photon is h ν.
If ν_0, is the threshold frequency (or λ_o is the threshold wavelength) of a metal. The work function of metal is $W_o = h\nu_o = \frac{hc}{\lambda_0}$ where, c is the speed of light.

If *m* is the mass of the emitted electron and v is its velocity, then kinetic energy of the emitted electron is $\frac{1}{2}mv^2$.

Hence, from the law of conservation of energy,
The energy imparted by the photon = Maximum kinetic energy of the emitted electron + Work function

$$h\nu = \frac{1}{2}m\text{v}_{\max}^2 + W_o$$

Thus, the maximum kinetic energy of the electron emitted $\frac{1}{2}m\text{v}_{\max}^2 = W_o - h\nu$

If the energy of the incident photon is just equal to the work function of the metal surface, the kinetic energy of the electron form the metal surface is zero.
In this condition, the frequency of the incident light is called the threshold frequency. Thus, when
$\nu = \nu_o$, then v = 0
$0 = h\nu_o - W_o$
$W_o = h\nu_o$

54) The following table gives the values of work function for a few photo sensitive metals

S.No.	Metal	Work Function (eV)
1.	Na	1.92
2.	K	2.15
3.	Mo	4.17

If each of these metals is exposed to radiations of wavelength 300 nm, which of them will not emit photo electrons and why ? [S.P.]

SOL: Wavelength of radiation used, $\lambda = 300 \times 10^{-9}$ m.

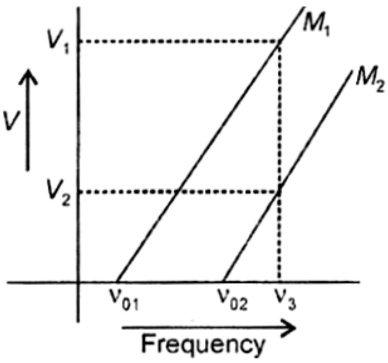
Energy of radiation corresponding to their wavelength is

$$E_n = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,\text{Js}\times3\times10^{8}\,\text{m/s}}{300\times10^{-9}\,\text{m}\times1.6\times10^{-19}}\text{eV} = 4.14\text{ eV}.$$

Energy corresponding to this wavelength is less than the value of work functions for Mo. Thus, Mo metals will not emit photoelectron.

55) By how much would the stopping potential for a given photosensitive surface go up if the frequency of the incident radiations were to be increased from 4×10^{15} Hz to 8×10^{15} Hz ?

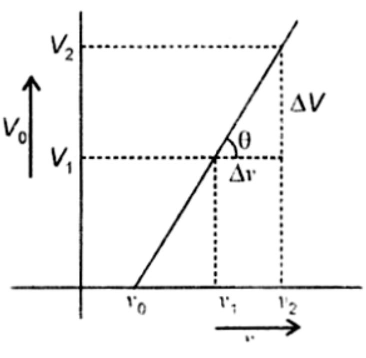
Given: $h = 6.4 \times 10^{34}$ J-s, $e = 1.6 \times 10^{-19}$C and $c = 3 \times 10^{8}$ ms^{-1}. [S.P.]

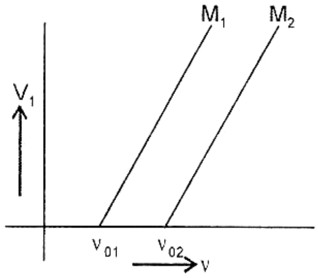


SOL:

$$V_0' - V_0 = \frac{h}{c}(\nu' - \nu) = \frac{6.4\times10^{-34}}{1.6\times10^{-19}}(8-4)\times10^{15} = 16\text{ volt.}$$

56) The given graphs show the variation of the stopping potential V_s with the frequency (ν) of the incident radiations for two different photosensitive materials M_1 and M_2.
(i) What are the values of work functions for M_1 and M_2 ?
(ii) The values of the stopping potential for M_1 and M_2 for a frequency $\nu_3(>\nu_{02})$ of the incident radiations are V_1, and V_2 respectively: Show that the slope of the lines equals
$\frac{V_1 - V_2}{\nu_{02} - \nu_{01}}$. **[S.P.]**





SOL: (i) Work function for material M_1 is $h\nu_{01}$ and work function for material M_2 is $h\nu_{02}$.

(ii) Observation depicted in the graph is given.

We have for material M_1 $eV_1 = h\nu_3 - h\nu_{01}$.....(i)

And for material M_2 $eV_2 = h\nu_3 - h\nu_{02}$.....(ii)

Subtracting equation (i) from equation (ii) we have

$e(V_1 - V_2) = h(\nu_{02} - \nu_{01})$

$\frac{h}{e} = \frac{V_1 - V_2}{\nu_{02} - \nu_{01}}$.....(iii)

For the single material for two different frequencies ν_1 and $\nu_2 > \nu_0$ we have stopping potentials V_1 and V_2 .

$eV_1 = h\nu_1 - h\nu_0$.....(iv)

$eV_2 = h\nu_2 - h\nu_0$.....(v)

(v)–(vi)

$e(V_2 - V_1) = h(\nu_2 - \nu_1)$

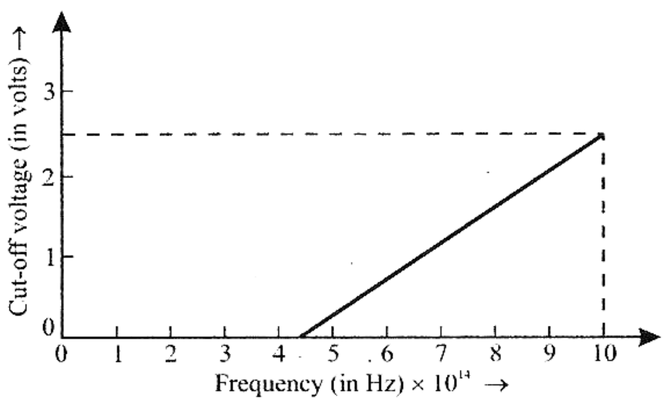
$\frac{h}{e} = \frac{(V_2 - V_1)}{(\nu_2 - \nu_1)}$...(vi)

Also $\tan\theta = \frac{\Delta V}{\Delta \nu}$ i.e. slope of the line from equation (vi) and equation (iii) we conclude that

$\frac{(V_2 - V_1)}{(\nu_{02} - \nu_{01})}$ is the slope of the either line.

57) For photoelectric effect in sodium, Fig. shows the plot of cut-off voltage versus frequency of incident radiation. Calculate :

(i) the threshold frequency. (ii) the work function for sodium. [CBSE D 95]



SOL: (i) From the given graph, threshold frequency is ν_3 = **4.5×10^{14}** Hz

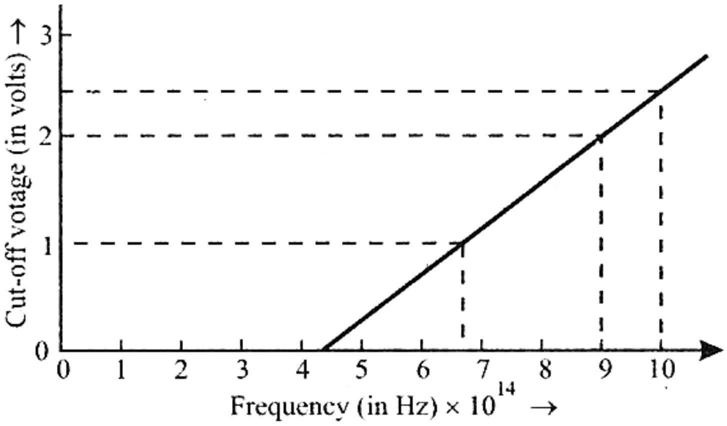
(ii) Work function of the metal is

$W_0 = h\,v = 6.6 \times 10^{34} \times 4.5 \times 10^{14}$

$= 2.97 \times 10^{-19}$ j = **1.86** eV.

58) Fig. shows the plot of cut-off voltage vs. frequency of radiation incident on a metal. Calculate

- **i) the threshold frequency, and**
- **ii) Planck's constant. [CBSE F 95]**



SOL: *(i)* From the given graph, the threshold frequency is ν_0 = 4.5 × **10^{14}** Hz

(ii) Using Einstein's photoelectric equation,

$$eV = h\nu - W_0$$

On differentiation, we get $e\,\Delta V = h\,\Delta\nu$.

$$h = \frac{\Delta V}{\Delta\nu} \cdot e = \frac{2.5 - 2}{(10 - 9) \times 10^{14}} \times 1.6 \times 10^{-19}$$

$$= \mathbf{8 \times 10^{-34}}\ \textbf{Js.}$$

PRACTICE QUESTIONS (Dual nature of matter and de-Broglie) Pg No 42

1) **Are matter waves electromagnetic ? Why?**

SOL: No. Electromagnetic waves are produced by accelerated charged particles, whereas matter waves are associated with a moving particle irrespective of charge on it.

2) **What is the momentum of a photon of frequency ν .**

SOL: $p = \frac{h}{\lambda} = \frac{h\nu}{c}$.

3) **Why are de-Broglie waves associated with a moving football not visible ?**

SOL: The wavelength $\lambda = \frac{h}{m\text{v}}$ associated with the moving football is too small to show any effect on the eye . Hence its wave nature is not visible.

4) **An electron and alpha particle have the same kinetic energy. How are the de-Broglie wavelength associated with them related? [Delhi 2008]**

SOL: de – Broglie wavelength , $\lambda = \frac{h}{\sqrt{2mK}}$ $[\because p = \sqrt{2mK}]$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

$$\frac{\lambda_e}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_e}}.$$

5) **Find the ratio of the de-Broglie wavelength, associated with**
   a) **protons, accelerated through a potential of 128 V and**
   b) **α -particles, accelerated through a potential of 64 V. [Delhi 2010C]**

SOL: de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}} \quad (\because K = qV)$$

$$\lambda \propto \frac{1}{\sqrt{mqV}}$$

where, m = mass of charge particle, q = charge, V = potential difference,

$\therefore$ Ratio of de Broglie wavelengths of proton and α -particle.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$$

$$= \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)}$$

Here $\frac{m_\alpha}{m_p} = 4, \frac{q_\alpha}{q_p} = 2, \frac{V_\alpha}{V_p} = \frac{64}{128} = \frac{1}{2}$

$\because$ α -particle is n times heavier than proton and it has double the charge than that of proton)

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{4 \times 2 \times \frac{1}{2}} = 2$$

$$\lambda_p : \lambda_\alpha = 2 : 1.$$

6) **The ratio between the de-Broglie wavelengths, associated with protons, accelerated through a potential of 512 V and α -particles, accelerated through a potential of x volt, is found to be one. Find the value of x.[Delhi 2010C]**

SOL: de Broglie wavelength of accelerated charge particle

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda \propto \frac{1}{\sqrt{mqV}}$$

Ratio of wavelengths of proton and α -particle.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)}$$

Here, $\frac{m_\alpha}{m_p} = 4, \frac{q_\alpha}{q_p} = 2, \frac{V_\alpha}{V_p} = \frac{X}{512}, \frac{\lambda_p}{\lambda_\alpha} = 1$

$$1 = \sqrt{4 \times 2 \times \left(\frac{x}{512}\right)}$$

$$1 = \frac{x}{64}$$

$$x = 64 \text{ V}$$

7) **Calculate the ratio of the accelerating potential required to accelerate**
   a) **a proton and**
   b) **an α -particle to have the same de-Broglie wavelength associated with them. [Delhi 2009C]**

SOL: de-Broglie matter wave equation for accelerating charge particle is given by

$\lambda = \frac{h}{\sqrt{2mqV}}$ where, h = Planck's constant m = mass of charge particle q = charge of charge particle

V = potential difference

Ratio of wavelengths of proton and α -particle.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)}$$

$\because$ $\frac{\lambda_p}{\lambda_\alpha} = 1, \frac{m_\alpha}{m_p} = 4, \frac{q_\alpha}{q_p} = 2, \frac{V_\alpha}{V_p} = ?$

$$1 = \sqrt{4 \times 2 \times \left(\frac{V_\alpha}{V_p}\right)}$$

$$1 = 8 \times \frac{V_\alpha}{V_p}$$

$$\frac{v_p}{v_\lambda} = 8$$

$$V_p : V_\alpha = 8 : 1.$$

8) **de-Broglie wavelength associated with an electron accelerated through a potential difference V is λ,. What will be its wavelength when the accelerating potential is increased to 4 V?[All India 2006]**

SOL: For electron beam, de Broglie wavelength

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å} \quad \text{for } V = 4 \text{ V}$$

$$\lambda = \frac{12.27}{\sqrt{4}} = 6.135 \text{ Å}.$$

9) **An electron is accelerated through a potential difference of 100 V. What is the de-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond. [Delhi 2010]**

SOL: Wavelength of accelerated electron beam from de-Broglie equation

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

for $V = 100$ V

$$\lambda = 1.227 \text{ Å}$$

This wavelength belong to the X-ray part of electromagnetic radiation.

10) **An electron is accelerated through a potential difference of 144 V. What is the de-Broglie wavelength associated with it? To which part of electromagnetic spectrum does this wavelength correspond? [Delhi 2010]**

SOL: Wavelength of accelerated electron beam from de-Broglie equation

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

V= 144 V

$\lambda = 1$ $\mathring{\mathbf{A}}$

This wavelength belong to X-ray part of electromagnetic spectrum

11) **An electron is accelerated through a potential difference of 64 V. What is the de-Broglie wavelength associated with it ? To which part of the electromagnetic spectrum does this value of wavelength correspond? [Delhi 2010]**

SOL: Wavelength of accelerated electron beam from de-Broglie equation

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

V= 64 V

$\lambda = 1.5$ $\mathring{\mathbf{A}}$

This wavelength belong to X-ray.

12) **If the potential difference used to accelerate electrons is tripled, by what factor does the de-Broglie wavelength of the electron beam change?**

SOL: The de Broglie wavelength of electrons accelerated through a p.d. of V volt is given by

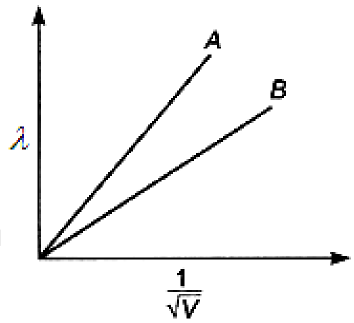
$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

If V is tripled , λ will decrease by a factor $\frac{1}{\sqrt{3}}$.

13) The two lines marked A and B in the given figure, show a plot of de-Broglie wavelength λ verses $\frac{1}{\sqrt{V}}$ where V is the accelerating potential, for two nuclei ^2_1H and ^3_1H.

a) What does the slope of the lines represent?

b) Identify which lines corresponded to these nuclei.



SOL: de Broglie wavelength of accelerating charge particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda\sqrt{V} = \frac{h}{\sqrt{2mq}} = \text{constant}$$

(a) The slope of the line represent $\frac{h}{\sqrt{2mq}}$ where h = Planck's constant, q and m = charge and mass of charge particle.

(b) $_1H^2$ and $_1H^3$ carry same charge (as they have same atomic number)

$$\lambda\sqrt{V} \propto \frac{1}{\sqrt{m}}$$

The lighter mass i.e., $_1H^2$ is represented by line of greater slope i.e., A and similarly $_1H^3$ by line *B.*

14) Derive an expression for the de Broglie wavelength associated with an electron accelerated through a potential V. Draw a schematic diagram of a localized-wave describing the wave nature of the moving electron. [Foreign 2009]

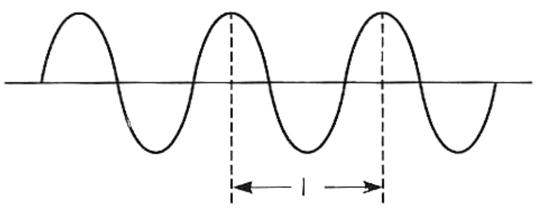
SOL: Let an electron beam is accelerated by potential difference V from the position of rest.

∴ Kinetic energy of the electron K = eV

Momentum of electron $= \sqrt{2mK}$

$p = \sqrt{2meV}$ where, m = mass of electron

∴ By de-Broglie matter wave equation



$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

Substituting $h = 6.62\times10^{-34}\,Js,\quad m = 9.1\times10^{-31}kg,\ and\ e = 1.6\times10^{-19}C$

We get $\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times V}}$

$$\lambda = \left(\frac{12.27\times10^{-10}}{\sqrt{V}}\right) = \frac{12.27}{\sqrt{V}}\,\overset{o}{\mathbf{A}}$$

A matter wave associated with an electron of definite momentum has single wavelength and extends all over space.

15) What is the de Broglie wavelength associated with an electron, accelerated through a potential difference of100 volts ? [NCERT ; CBSE 06]

SOL: Accelerating potential, V =100 V.

The de Broglie wavelength is $\lambda = \frac{1.227}{\sqrt{V}} nm = \frac{1.227}{\sqrt{100}} nm \approx$ 0.123 nm.

This wavelength is of the order of X-ray wavelengths.

16) Calculate the ratio of the accelerating potential required to accelerate

a) a deuteron and

b) an α -particle to have the same de-Broglie wavelength associated with them. (Given, mass of deuteron = 3.2 × 10^{-27} kg) [Delhi 2009C]

17) Crystal diffraction experiments can be performed either by using electrons accelerated through appropriate voltage or by using X-rays. If the wavelength of these probes (electrons or X-rays) is 1 $\overset{o}{\mathbf{A}}$, estimate which of the two has greater energy. [All India 2009]

SOL: For an accelerated electron beam, the de-Broglie matter wave equation states that

$$\lambda = \frac{h}{\sqrt{2emV}} = \frac{h}{\sqrt{2mK}}$$

$$K = \frac{h^2}{2m\lambda^2} \qquad \ldots(i)$$

For X-ray photon of same wavelength λ =1 $\overset{o}{\mathbf{A}}$.

$$E' = h\nu = \frac{hc}{\lambda} \qquad \ldots(ii)$$

$$\frac{K}{E'} = \frac{h^2}{2m\lambda^2}\bigg/\frac{hc}{\lambda}$$

$$\frac{K}{E'} = \frac{h^2}{2m\lambda^2}\times\frac{\lambda}{hc} = \frac{h}{2mc\lambda}$$

$$\frac{K}{E'} = \frac{h}{2mc\lambda}$$

where, $h = 6.6\times10^{-34}$ J-s,

$m = 9.1\times10^{-31}$ kg

$c = 3\times10^{8}$ m/s,

$\lambda = 1\text{Å} = 1\times10^{-10}$ m

$\lambda = 1\text{Å} = 1\times10^{-10}$ m

$$\frac{K}{E'} = \frac{11}{911} < 1$$

Energy possess by X-ray is more than electron.

18) For what kinetic energy a neutron, will the associated de-Broglie wavelength be 1.32×10^{-10} m? [All India 2008]

SOL: From de Broglie matter wave equation

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \qquad (\because\ p = \sqrt{2mK})$$

$$\Rightarrow \qquad K = \frac{h^2}{2m\lambda^2} \qquad \ldots(i)$$

where, $m = 1.66 \times 10^{-27}$ kg

$\lambda = 1.32 \times 10^{-10}$ m

$h = 6.63 \times 10^{-37}$ J-s

$$\therefore \quad K = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.66 \times 10^{-27} \times (1.32 \times 10^{-10})^2}$$

$$K = 7.5 \times 10^{-21} \text{ J.}$$

19) (a) Determine the de-Broglie wavelength of a proton whose kinetic energy is equal to the rest mass energy of an electron. Mass of [All India 2011C][Comptt. Delhi 2011]

a) proton 1836 times that of electron.

b) **In which region of electromagnetic spectrum does this wavelength lie?**

SOL: de Broglie matter wave equation is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$p = \sqrt{2mK}$$

where, m = mass of proton,
K = kinetic energy of proton.
According to question, kinetic energy of proton
$K = m_e c^2$ (Einstein's mass-energy relation)

$$\lambda = \frac{h}{\sqrt{2m(m_e c^2)}}$$

$$\lambda = \frac{h}{\sqrt{2}\, c\sqrt{m \cdot m_e}}$$

$$\lambda = \frac{h}{\sqrt{2}c \times (m_e)\sqrt{1836}} \quad k\,(\because\ m = 1836 m_e)$$

$$\lambda = \frac{6.63 \times 10^{-34}}{1.414 \times (3 \times 10^8) \times 9.1 \times 10^{-31} \times 42.8}$$

$$\lambda = 4 \times 10^{-14} \text{ m}$$

(b)This region of electromagnetic spectrum is X-ray.

20) (a) The mass of a principle moving with velocity 5×10^6 m/s has de-Broglie wavelength associated with it to be 0.135 nm. Calculate its mass.

(b) In which region of the electromagnetic spectrum does this wavelength lie? [All India 2011]

SOL: (a) From de Broglie matter wave eauation

$$\lambda = \frac{h}{mv}$$

$$m = \frac{h}{\lambda v}$$

Here, $\lambda = 0.135 \times 10^{-9}$ m,

$v = 5 \times 10^6$ m/s.

$$\therefore \quad m = \frac{6.63 \times 10^{-34}}{0.135 \times 10^{-9} \times 5 \times 10^6}$$

$$= 9.82 \times 10^{-31} \text{ kg}$$

(b) This wavelength 0.135 nm falls in the region of X-ray of electromagnetic spectrum.

21) An electron and alpha particle have the same de-Broglie wavelength with them. How are their kinetic energies related to each other? [Delhi 2008]

SOL: From de Broglie wave equation

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$mK = \frac{h^2}{2\lambda^2} = \text{constant}$$

(for given wavelength)

$$K \propto \frac{1}{m}$$

$$\frac{K_e}{K_\alpha} = \frac{m_\alpha}{m_e}$$

where m_e and m_α are masses of electron and α -particles.

22) An electron and a proton are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it and (ii) less momentum ? Justify your answer.

SOL: de - Broglie wave length associated with the matter of mass m and charge q accelerated with a potential difference V is given by $\lambda = \frac{h}{\sqrt{2mqV}}$. for an electron q = e and for a proton q = e

$$\therefore \quad \lambda \propto \frac{1}{\sqrt{m}} \qquad \text{[as } V \text{ and } q \text{ are same for both]}$$

$\because \; m_P > m_e \qquad \therefore \; \lambda_p < \lambda_e$.

(ii) Further, $\lambda = \frac{h}{mv}$

or $$mv = \frac{h}{\lambda}$$

$$p = \frac{h}{\lambda} \qquad \text{[where, } p = mv\text{]}$$

$$p \propto \frac{1}{\lambda}$$

$\because \; \lambda_P < \lambda_e \qquad \therefore \; p_p > P_e$

23) **An electron and a proton are accelerated through the same potential. Which one of the two has**

a) **greater value of de-Broglie wavelength associate with it and**

b) **less momentum? Justify your answer. [Delhi 2009]**

SOL: From de-Broglie matter wave equation

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

as $p = \sqrt{2mK}$ and $K = qV$

$$\lambda = \frac{h}{\sqrt{2mqV}} \quad \ldots(i)$$

where, signs are as usual for given accelerating potential.

$$\lambda \propto \frac{1}{\sqrt{mq}}$$

Ratio of wavelengths of electron and proton.

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\left(\frac{m_p}{m_e}\right)\left(\frac{q_p}{q_e}\right)}$$

Ratio of mass of proton and electron

$$\frac{m_p}{m_e} = 1836 \text{ (constant)}$$

$$\frac{q_p}{q_e} = 1$$

(Both electron and proton have same charge)

$$\frac{\lambda_e}{\lambda_p} = \sqrt{1836 \times 1}$$

$$\lambda_e \approx 42.8\lambda_p \text{nearly}$$

Electron have greater wavelength associated with it than that of proton.

(b) $\lambda = \frac{h}{p}$ (de-Broglie matter wave equation)

$$p = \frac{h}{\lambda}$$

$$p \propto \frac{1}{\lambda}$$

$$\frac{p_e}{p_p} = \frac{\lambda_p}{\lambda_e}$$

But from Eq. (i), $\frac{\lambda_p}{\lambda_e} = \frac{1}{42.8}$

$$\frac{p_e}{p_p} = \frac{\lambda_p}{\lambda_e} = \frac{1}{42.8}$$

Momemtum of proton is nearly 42.8 times to that of momentum of electron.

24) **An electron, α -particle and a proton have the same de-Broglie wavelengths. Which of these particle has**

a) **minimum kinetic energy,**

b) maximum kinetic energy and why? In what way has the wave nature of electron been exploited in electron microscope? [Foreign 2007] [Comptt. Delhi 2017]

SOL: de Broglie matter wave equation

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \qquad \left(\because K = \frac{p^2}{2m}\right)$$

where, *K* is kinetic energy and *m* is mass of particle.

$$K = \frac{h^2}{2m\lambda^2}$$

(for same wavelength λ)

$$K \propto \frac{1}{m}$$

$$K_e : K_\alpha : K_p = \frac{1}{m_e} : \frac{1}{m_\alpha} : \frac{1}{m_p}$$

where m_e, m_p and m_α are masses of electron, proton and α -particle respectively.

Also, K_e, , K_α and K_p are their respective kinetic energies.

$$m_\alpha > m_p > m_e$$

$$m_\alpha m_p > m_e m_\alpha > m_e m_p$$

$$K_e > K_p > K_\alpha \qquad \ldots \text{(ii)}$$

(a) α -particle possess minimum KE

(b) Electron has maximum KE.

The magnifying power of an electron microscope is inversely related to wavelength of radiation used. Smaller wavelength of electron beam in comparison to visible light increases the magnifying power of microscope.

Electrons accelerated through a high voltage (say , 100kV) have (de Broglie) wavelength as small as 0.04 Å , about 1.25×10^5 times smaller than that of visible light (5000 Å) . Hence the resolving power of microscope using electron –beam is 1.25×10^5 times higher than an optical microscope.

25) **A proton and an alpha particle, both initially at rest, are (suitably) accelerated so as to have the same kinetic energy. What is the ratio of their de-Broglie wavelengths? [CBSE Sample Paper]**

SOL: Let mass of a proton *(m_p) = m* , ∴ Mass of an alpha particle (m_α) = 4 m

Given, $K_p = K_\alpha$

de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mK}}$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_p K_p}} \times \frac{\sqrt{2m_\alpha K_\alpha}}{h}$$

$$= \sqrt{\frac{m_\alpha}{m_p}} \qquad (\because K_p = K_\alpha)$$

$$= \sqrt{\frac{4m}{m}} = \frac{2}{1}$$

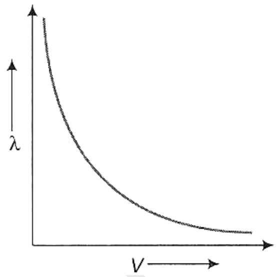
26) The de Broglie wavelengths, associated with a proton and a neutron, are found to be equal. Which of the two has a higher value for kinetic energy ? [S.P.]

SOL: As $KE \propto \frac{1}{m}$ for same value of de-Broglie wavelength, $KE_P > KE_n$ $(\because m_n > m_p)$

27) Show, on a graph, the nature of variation, of the (associated) de-Broglie wavelength (λ_B), with the accelerating potential (V), for an electron initially at rest. [CBSE Sample Paper]

SOL: de-Broglie wavelength associated with an electron is given by

$$\lambda_B = \frac{h}{\sqrt{2m\,eV}} = \frac{12.27}{\sqrt{V}}\,\text{Å} \quad \therefore \quad \lambda_B \propto \frac{1}{\sqrt{V}}$$



A graph for de-Broglie wavelength associated with an electron is shown in figure.

28) The most probable kinetic energy of thermal neutrons at a temperature of T kelvin, may be taken as equal to *kT*, where *k* is Boltzmann constant. Taking the mass of a neutron and its associated de-Broglie wavelength as *m* and λ_B respectively, state the dependence of λ_B on m and T. [CBSE Sample Paper]

SOL:

de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mE}}$

$$\lambda_B = \frac{h}{\sqrt{2mkT}} \qquad [\because E = kT]$$

$$\Rightarrow \quad \lambda_B \propto \frac{1}{\sqrt{mT}} \qquad [\because h \text{ and } k \text{ are constant}]$$

where, m is the mass of neutrons.

29) Calculate the de-Broglie wavelength of (i) an electron (in the hydrogen atom) moving with a speed of $\frac{1}{100}$ of the speed of light in vacuum and (ii) a ball of radius 5mm and mass 3×10^{-2} kg moving with a speed of 100 m/s Hence show that the wave nature of matter is important at the atomic level but is not really relevant at the macroscopic level. [S.P.]

SOL:

(*i*) de-Broglie wavelength, $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{mv}$$

Here $m = 9.11 \times 10^{-31}$ kg

$$v = \frac{c}{100} = \frac{3\times10^8}{100} = 3 \times 10^6 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$\lambda = \frac{6.63\times10^{-34}}{9.11\times10^{-31}\times3\times10^6} = 2.43 \times 10^{-10} \text{ m}$$

$$\lambda = 2.43\ \text{Å} = 0.243 \times 10^{-9}\ \text{m}$$

(*ii*) mass of the ball, $m' = 3 \times 10^{-2}$ kg

speed of the ball, $u' = 100\ \text{ms}^{-1}$

de-Broglie wavelength $\lambda' = \frac{h}{p'} = \frac{6.63 \times 10^{-34}}{3 \times 10^{-2} \times 100}$

$$\lambda' = 2.21 \times 10^{-34}\ \text{m}$$

It is clear from the results that wavelength of the ball is negligible in comparison to the wavelength of the electron *i.e.*, at macroscopic level wave nature of matter is not relevant.

30) For what kinetic energy of neutron will the associated de-Broglie wavelength be 1.32 ×10^{-10} m? the mass of neutron is 1.675×10^{-27} kg and h=6.626×10^{-34} Js.

SOL:

The de Broglie wavelength of a particle of mass m and kinetic energy K is given by

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

From this, we have

$$K = \frac{h^2}{2m\lambda^2}.$$

Substituting the given values, we have

$$K = \frac{(6{\cdot}626 \times 10^{-34}\ \text{J s})^2}{2 \times (1{\cdot}675 \times 10^{-27}\ \text{kg}) \times (1{\cdot}32 \times 10^{-10}\ \text{m})^2} = 7{\cdot}52 \times 10^{-21}\ \text{J.}$$

31) An electromagnetic wave of wavelength λ_1 is incident on a photosensitive surface of negligible work function. If the photoelectrons emitted from this surface have the de-Broglie wavelength λ, prove that $\lambda = \left(\frac{2mc}{h}\right)\lambda_1^2$ [Delhi 2008]

SOL: For negligible work function, by Einstein's photoelectric equation

$$\frac{hc}{\lambda} = \text{KE}_{max} + 0\ (\phi \text{ is very small})$$

$$\frac{hc}{\lambda} = \frac{p^2}{2m} \qquad \left(\text{KE} = \frac{p^2}{2m}\right)$$

where, p = momentum of electron,
m = mass of electron.

$$p = \sqrt{\frac{2mhc}{\lambda}} \qquad \ldots(i)$$

$\therefore$ de-Broglie wavelength $\lambda_1 = \frac{h}{p}$

$$\lambda_1 = \frac{h}{\sqrt{\frac{2mhc}{\lambda}}} \qquad \text{[From Eq. (i)]}$$

$$\lambda_1 = \sqrt{\frac{h\lambda}{2mc}}$$

Squaring both sides, we get

$$\lambda_1^2 = \frac{h\lambda}{2mc} \quad \Rightarrow \quad \lambda = \left(\frac{2mc}{\lambda}\right)\lambda_1^2.$$

32) An electron, an alpha-particle and a proton have same kinetic energy. Which one of these particles has the shortest de-Broglie wavelength?

SOL:

The de-Broglie wavelength of a particle of mass m and kinetic energy K is given by

$$\lambda = \frac{h}{\sqrt{2mK}} \quad \text{or} \quad \lambda \propto \frac{1}{\sqrt{m}}. \qquad \text{(for same } K\text{)}$$

Since the mass of α-particle is greatest, the wavelength of α-particle is shortest.

33) Calculate the de Broglie wavelength associated with an electron of energy 200 eV. What will be the change in this wavelength if the accelerating potential is increased to four times its earlier value ? [CBSE Sample Paper 03]

SOL: Here $K = 200$ eV $= 200 \times 1.6 \times 10^{-19}$ J, m $= 9.1 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ Js

de-Broglie wavelength of the electron,

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 200 \times 1.6 \times 10^{-19}}}$$

$= 0.8648 \times 10^{-10}$m = **0.8648** Å

For an electron accelerated through potential difference V,

$$\lambda = \frac{h}{\sqrt{2\,meV}}$$

i.e., $$\lambda \propto \frac{1}{\sqrt{V}}$$

$$\therefore \quad \frac{\lambda'}{\lambda} = \sqrt{\frac{V}{V'}} = \sqrt{\frac{V}{4V}} = \frac{1}{2}$$

or $$\lambda' = \frac{\lambda}{2}.$$

Thus wavelength would become half of its initial value.

34) A particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813 × 10^{-4}. Calculate the particle's mass and identify the particle. Mass of electron =9.11 × 10^{-31} kg [NCERT][Comptt. Outside Delhi 2011]

SOL: de Broglie wavelengths of the particle and the electron are

$$\lambda_p = \frac{h}{m_p v_p} \quad \text{and} \quad \lambda_e = \frac{h}{m_e v_e}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_e} = \frac{m_e v_e}{m_p v_p} \quad \text{or} \quad \frac{m_p}{m_e} = \frac{\lambda_e}{\lambda_p} \cdot \frac{v_e}{v_p}$$

Given $v_p = 3\,v_e$ and $\frac{\lambda_p}{\lambda_e} = 1.813 \times 10^{-4}$

$$\therefore \quad \frac{m_p}{m_e} = \frac{1}{1.813 \times 10^{-4}} \times \frac{1}{3}$$

$$m_p = \frac{m_e}{3 \times 1.813 \times 10^{-4}}$$

$$= \frac{9.11 \times 10^{-31}}{3 \times 1.813 \times 10^{-4}} \text{ kg}$$

$= 1.675 \times 10^{-27}$ kg.

This is the mass of a neutron. So the given particle is a neutron.

35) Show that the wavelength of electro-magnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

SOL: For a photon, de Broglie wavelength, $\lambda = \frac{h}{p}$.

For an electromagnetic radiation of frequency ν and wavelength $\lambda' = \frac{c}{\nu}$,

Momentum,

$$p = \frac{E}{C} = \frac{h\nu}{c}$$

or $$p = \frac{h}{c} \cdot \frac{c}{\lambda'} = \frac{h}{\lambda'}$$

Then, $\lambda' = \frac{h}{p} = \lambda$

Thus the wavelength λ' of the electro-magnetic radiation is the same as the de Broglie wavelength λ of the photon.

QUESTIONS (STRUCTURE OF ATOM) Pg No-72

1) **Define ionization energy. What is its value for a hydrogen atoms? [All India 2010]**

SOL: Ionisation Energy The minimum amount of energy required to excite an electron from the ground state of the atom, is known as ionisation energy.

Ionisation energy for hydrogen atom =13.6 eV.

2) **What is the diameter of hydrogen atom?**

SOL: It is equal to the diameter of the first orbit of H_2 - atom *i.e.* 1.06 $\mathring{A}$.

3) **Describe Rutherford's atomic model.**

SOL: The whole mass and positive charge of the atom is concentrated in a small central core, called nucleus. Electrons having equal negative charge revolve around the nucleus.

4) **What are the drawbacks of Rutherford atom model?**

Or **Give two drawbacks of Rutherford's atom model.**

SOL: 1. It could not explain the stability of the atom. The electrons revolving around the nucleus are continuously accelerated. Since an accelerated charge emits energy, the radius of the circular path of a revolving electron should go on decreasing and ultimately it should fall into the nucleus.

2. It could not explain the line spectrum of the atoms, like hydrogen atom. According to Rutherford's atom model, as the electrons can revolve in orbits of all possible radii, an atom should emit continuous energy spectrum.

5) **Why did Thomson's atomic model fail ?**

SOL: As in Rutherford's α -scattering experiment, it (Thomson's model) could not explain the scattering of α -particles through large angles.

6) **Define impact parameter.**

SOL: Impact parameter of the alpha particle is defined as the perpendicular distance of the velocity vector of the alpha particle from the centre of the nucleus, when it is far away from the atom.

7) **Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its**
   a) **second permitted energy level to the first level and**
   b) **the highest permitted energy level to the first permitted level. [All India 2010]**

SOL: (a) Since, the second permitted energy level to the first level = $E_2 - E_1$ = Energy of photon released = (– 3.4 eV) – (–13.6 eV) =10.2 eV.

(b) The highest permitted energy level to the first permitted level = $E_\infty - E_1$= 0 –(–13.6) =13.6 eV

Ratio of energies of photon $\frac{10.2}{13.6} = \frac{3}{4} = 3:4$.

8) **The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of electron in this state? [All India 2010]**

SOL: Given, total ground state (E) = (–13.6 eV)

Kinetic energy E_k = –E = – (–13.6 eV) =13.6 eV

Potential energy E_p = 2 (E) =2 × (–13.6) = – 27.2 eV

9) The total energy of an electron in the first excited state of the hydrogen atom is about –3.4eV. What is the potential energy of the electron in this state ?

SOL: The potential energy (E_p) of the electron in an orbit is equal to twice its total energy (E).

Potential energy E_p = 2 (E) =2 × (–3.4) = – 6.8 eV

10) The total energy of an electron in the first excited state of the hydrogen atom is about 3.4eV, What is the kinetic energy of the electron in this state ?

SOL: The kinetic energy (E_k) of the electron in an orbit is equal to negative of its total energy (E).

$E_k = -E = -(-3{\cdot}4) = 3{\cdot}4$eV

11) What is the ratio of radii of the orbits corresponding to first excited state and ground state in a hydrogen atoms? [Delhi 2010]

SOL: For first excited state *n* = 2 **;** Ground state occurs for n =1

$$r_n = r_1 n^2$$

$$r \propto n^2$$

$$\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{1}\right)^2$$

$$\therefore \quad r_1 : r_2 = 4:1$$

where, r_1, and r_2 are radii corresponding to first excited state and ground state of the atom.

12) The radius of innermost electron orbit of a hydrogen atom is 5.3× 10^{-11} m. What is the radius of orbit in the second excited state?

SOL: The radius of atom whose principal quantum number is *n*, is given by

$$r = n^2 r_0 = 5.3 \times 10^{-11}\,\text{m}$$

where, r_o = radius of innermost electron orbit .

For second excited state, *n* = 3.

$$\therefore \quad r = 3^2 \times r_0 = 9 \times 5.3 \times 10^{-11}$$

$$r = 4.77 \times 10^{-10}\ \text{m.}$$

13) In the ground state of hydrogen atom, its Bohr radius is given as 5.3 x 10^{-11} m. The atom is excited such that the radius becomes 21.2 x 10^{-11} m. Find (i) the value of the principal quantum number and (ii) the total energy of the atom in this excited state.

SOL:

(i) $r = r_0 n^2$

$21.2\text{x}10^{-11} = 5.3\text{x}10^{-11}\ n^2$ implies n = 2

(ii) $E = \dfrac{-13.6eV}{n^2}$

$= \dfrac{-13.6eV}{2^2}$ =-3.4eV

14) (a)Write the expression for Bohr's radius in hydrogen atom. [Delhi 2010]

SOL: Expression for Bohr's radius in hydrogen atom,

$$r = 4\pi\varepsilon_0 \cdot \frac{n^2h^2}{4\pi^2 me^2} \qquad (\textbf{Bohr's Radius})$$

Where , n = principal quantum number,
m = mass of electron

$$k = \frac{1}{4\pi\varepsilon} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

Z = atomic number of atom =1
h = Planck's constant

(b)An electron revolves in a circular orbit around a nucleus of charge Z *e*. How is the electron velocity related to the radius of its orbit.

SOL: The relation between electron velocity and radius of the orbit is given by

$$mvr = n\frac{h}{2\pi}$$

$$v = \frac{nh}{2\pi mr}$$

$$r = 4\pi\varepsilon_0 \cdot \frac{n^2h^2}{4\pi^2 me^2} \qquad (\textbf{Bohr's Radius})$$

15) State Bohr's postulate for the 'permitted orbits' for the electron in a hydrogen atom. Use this postulate to prove that the circumference of the n^{th} permitted orbit for the electron can contain exactly *'n'* wave lengths of the de-Broglie wavelength associated with the electron in that orbit. [S.P.]

SOL: According to postulate for permitted orbits "Electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$ i.e.

$L = \frac{nh}{2\pi}$ Where h is Planck's constant.

Consider an electron in n^{th} permitted orbit revolving with speed v_n and having orbital radius r_n.
According to Bohr's postulate.

$$m\, v_n\, r_n = \frac{nh}{2\pi}$$

$$2\pi r_n = \frac{nh}{mv_n}$$

$$\frac{h}{mv_n} = \lambda (\text{de-Broglie wavelength})$$

$$2\pi r_n = n\lambda \quad ; \qquad n = 1, 2, 3, ...$$

Thus, it is clear that circumference of the orbit is equal to n wave lengths of the de-Broglie wavelength associated with the electron in the orbit.

16) In the Rutherford scattering experiment, the distance of closest approach for an α -particle is d_o. If α -particle is replaced by a proton, how much kinetic energy in comparison to α -particle will it require to have the same distance of closest approach d_o? [Foreign 2009]

SOL: $\because$ For given distance of closest approach Kinetic energy $\propto$ Z (atomic number)

$$\frac{K_{proton}}{K_\alpha} = \frac{Z_{proton}}{Z_\alpha} = \frac{1}{2}$$

$$K_{proton} : K_\alpha = 1 : 2$$

17) **In hydrogen atom, if the electron is replaced by a particle which is 200 times heavier but has the same charge, how would its radius change? [Foreign 2008]**

SOL: Radius of hydrogen atom $\propto \frac{1}{m}$ **;** where, m = mass of particle

$$\frac{r_2}{r_1} = \frac{m_1}{m_2} = \frac{m_1}{200m_1} = \frac{1}{200}$$

($\because$ The new particle is 200 time heavier)

$$r_2 = \frac{1}{200} r_1$$

New radius would become $\frac{1}{200}$ times to that of original radius.

18) **The energy of the electron in the ground state of hydrogen atom is –13.6 eV.**

a) **What does the negative sign signify?**

b) **How much energy is required to take an electron in this atom from the ground state to the first excited state? [Foreign 2009]**

SOL: (a) The negative sign imply that electrons are bound to the nucleus by means of electrostatic force of attraction.

(b) Energy of electron in nth orbit of hydrogen atom

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

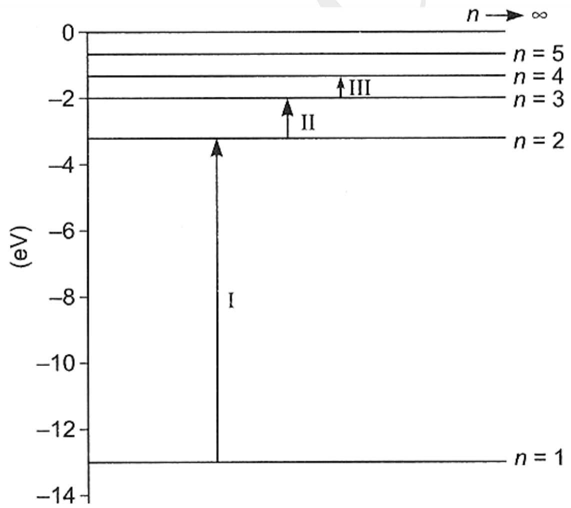
For first excited state $n = 2$

$$E_2 = -\frac{13.6}{2^2} \text{ eV}$$

$$= -\frac{13.6}{4} = -3.4 \text{ eV.}$$

– 3.4 eV energy is required

19) **Photons, with a continuous range of frequencies, are made to pass through a sample of rarefied hydrogen. The transitions, shown here, indicate three of the spectral absorption lines in the continuous spectrum.**



(a) Identify the spectral series, of the hydrogen emission spectrum, to which each of these three lines correspond.

(b) Which of these lines corresponds to the absorption of radiation of maximum wavelength? [Delhi 2009C]

SOL: (a) Lyman series is obtained when an electron jumps from the first orbit to any outer orbit. So, I spectrum represent Lyman series because in this spectrum, electron jumps from n =1 to $n = 2$.

Balmar series is obtained when an electorn jumps from the second orbit to any outer orbit. So, II spectrum represent Balmar series because in this spectrum, electron jumps from n = 2, to $n = 3$.

Similarly, in Paschen series, electron jumps from the third orbit to any outer orbit. So, III spectrum represent Paschen series.

(b) Spectral lines III *i.e.,* Paschen series corresponds to the absorption of radiation of maximum wavelength.

20) **Name the series of hydrogen spectrum, which has least wavelength.**

Ans. Lyman series.

21) **Name the series of hydrogen spectrum lying in the ultra-violet region.**

Ans. Lyman *series* lies in ultraviolet region.

22) Calculate the energy required to excite an electron from first orbit of the hydrogen atom to the third orbit ?

SOL: Energy of electron in the first orbit of the H- atom,

$$E_1 = -13{\cdot}6 \text{ eV}$$

and energy of electron in the third orbit of the H- atom,

$$E_3 = -\frac{13{\cdot}6}{3^2} = -1{\cdot}51 \text{ eV}$$

Therefore, energy required to excite an electron from first orbit of the H- atom to the third orbit,

$$E = E_3 - E_1$$
$$= -1{\cdot}51 - (-13{\cdot}6) = \mathbf{12{\cdot}09\ eV}$$

23) What is the maximum possible number of spectral lines observed, when the hydrogen atom is in its second excited state ? justify your answer. Calculate ratio of the maximum and minimum wavelengths of the radiations emitted in this process.

Ans. When the electron is in the second excited state ($n = 3$) of the hydrogen atom, the following transitions can take place

From the state $n = 3$ to $n = 2$ and $n = 1$ (two transitions)

and from the state n = 2 to n =1 (one transition)

Thus, the maximum possible number of spectral lines observed will be *three.*

The energy emitted is minimum, when the transition of electron takes place from the state $n = 2$ to n_1. The energy emitted is given by

$E_{min} = E_2 - E_l \quad = -3.4 - (-13{\cdot}6) = 10{\cdot}2$ eV

Likewise, the wavelength emitted will be maximum. The energy emitted is maximum, when the transition of electron takes place from the state $n = 3$ to n_1. The energy emitted is given by

$E_{max} = E_3 - E_l$

$$= -1.51 - (-13.6) = 12.09 \text{ eV}$$

Likewise, the wavelength emitted will be minimum.

Hence, ratio of the maximum and minimum wavelengths of the radiations emitted,

$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{hc/E_{min}}{hc/E_{max}} = \frac{E_{max}}{E_{min}} = \frac{12.09}{10.2} = \mathbf{1.185}$$

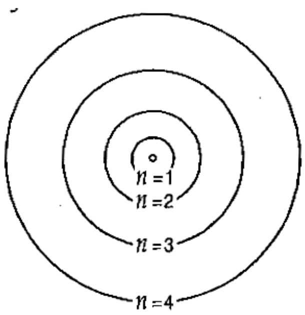
24) Calculate the radius of the smallest orbit of H- atom.

SOL: $r = 4\pi\varepsilon_0 . \frac{n^2h^2}{4\pi^2 me^2}$ Substituting h= 6.6×10^{-34} J .s , m = 9.1×10^{-31} kg, $4\pi\varepsilon_0 = \frac{1}{9\times10^9}$; e = 1.6×10^{-19} C

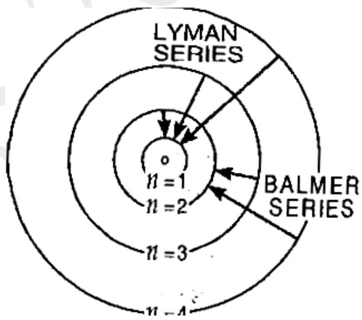
$$r = n^2 \times 5.29\times10^{-11} m$$

$$r = 0.529\,\mathring{A} \simeq 0.53\,\mathring{A}$$

25) In the Fig. for the stationery orbits of the hydrogen atom, mark the transitions representing the Balmer and Lyman series.



SOL:



26) The ground state energy of hydrogen atom is –13.6 eV. The photon emitted during the transition of electron from *n* = 2 to *n* =1 state, is incident on the photosensitive material of unknown work function. The photoelectrons are emitted from materials with a maximum kinetic energy of 8 eV. Calculate the threshold wavelength of the material used. [Foreign 2008]

SOL: Energy of electron in nth orbit of hydrogen atom

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

For $n = 1$,

$$E_1 = -13.6 \text{ eV}$$

For $n = 2$,

$$E_2 = -\frac{13.6}{4} = -3.4 \text{ eV}$$

Energy of photon released = $E_2 - E_1$ = (–3.4)–(–13.6)=10.2eV = $h\nu$

Also $K_{max} = 8eV$

According to Einstein equation $K_{max} = h\nu - W_o$

8eV = 10.2 eV – W_o

Work function W_o = 2.2 eV

Threshold wavelength $\lambda = \frac{1242\ eV - nm}{W_o} = \frac{1242\ eV - nm}{2.2eV} = 564.5nm$

27) In Bohr's theory of hydrogen atoms, calculate the energy of the photon emitted during a transition of the electron from the first excited state to its ground state. Write in which region of the electromagnetic spectrum this transition lies. Given Rydberg constant *R* = 1.03×10^7 m^{-1} .[Delhi 2008C]

SOL: Energy of electron in nth orbit of hydrogen atom

$$E_n = -\frac{13.6}{n^2}\ eV$$

For ground state, $n = 1$

$$E_1 = -\frac{13.6}{1^2} = -13.6\ eV$$

For Ist excited state, $n = 2$

$$E_2 = -\frac{13.6}{2^2} = -3.4\ eV$$

$\therefore$ Energy of photon released

$$E_2 - E_1 = (-3.4\ eV) - (-13.6\ eV) = 10.2\ eV$$

For Lyman series

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right], \quad \text{where } n = 2, 3, 4, \ldots$$

Here, $n = 2$

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R$$

$$\frac{1}{\lambda} = \frac{3}{4}R$$

$$\Rightarrow \quad \lambda = \frac{4}{3R}$$

$$\lambda = \frac{4}{3 \times 1.03 \times 10^7}$$

$$= 1.3 \times 10^{-7}\ m$$

$$\lambda = 1300\ Å$$

This wavelength falls in the region of Lyman series.

28) The energy of the electron, the hydrogen atom, is known to be expressible in the form

$E_n = \frac{-13.6eV}{n^2}$ **, (where, *n* =1 2, 3 ...) Use this expression to show that the**

(a) electron in the hydrogen atom cannot have an energy of – 2 eV.

(b) spacing between the lines (consecutive energy levels) within the given set of the observed hydrogen spectrum decreases as *n* increases. [All India 2008C]

SOL:

For $E_n = -2\text{ eV}$

$$-2\text{ eV} = \frac{-13.6}{n^2}\text{ eV}$$

$$13.6 = 2n^2$$

$$n^2 = 6.8$$

n is necessarily non-integral value whereas it should be integer to satisfy quantization condition. Therefore, – 2 eV energy of electron is not possible.

(b) Energy of the electron, $E_n = \frac{-13.6}{n^2}$

For $n = 1$, $E_1 = \frac{13.6}{1} = -13.6\text{ eV}$

For $n = 2$, $E_2 = \frac{13.6}{(2)^2} = -3.4\text{ eV}$

For $n = 3$, $E_3 = \frac{13.6}{3^2} = -1.51\text{eV}$

For $n = 4$, $E_4 = \frac{13.6}{(4)^2} = -0.85\text{ eV}$

$$E_n - E_{n-1} < E_{n-1} - E_{n-2} \qquad \forall\, n \in N$$

So, from above result we can say that spacing between spectral lines decreases.

29) The energy of the electron, in the hydrogen atom, is known to be expressible in the form

$E_n = \frac{-13.6eV}{n^2}$ **(where *n* = 1, 2, 3...) Use this expression to show that the**

(a) electron in the hydrogen atom cannot have an energy of – 6.8 eV.

(b) spacing between the lines (consecutive energy levels) within the given set of the observed hydrogen spectrum decreases as *n* increases. [All India 2008C]

30) The ground state energy of hydrogen atom is – 13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to – 3.4 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?[All India 2012]

31) State the basic assumption of the Rutherford model of the atom. Explain, in brief why this model cannot account for the stability of an atom? [Delhi 2010C]

SOL: Basic assumptions of Rutherford atomic model are given below

1) Atom consists of small central core, called atomic nucleus in which whole mass and positive charge is assumed to be concentrated.

2) The size of the nucleus is much smaller than size of the atom.

3) The nucleus is surrounded by electrons. Atoms are electrically neutral as total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.

4) Electrons revolves around the nucleus in various circular orbits and necessary centripetal force is provided by electrostatic force of attraction between positively charged nucleus and negatively charged electrons.

Stability of atom :When an electron revolves around the nucleus, then it radiate electromagnetic energy and hence, radius of orbit of electron decreases gradually. Thus, electron revolve on spiral path of decreasing radius and finally, it should fall into nucleus, but this does not happen. Thus, Rutherford atomic model cannot account for stability of atom.

32) State any two postulates of Bohr's theory of hydrogen atom. What is the maximum possible number of spectral lines observed when the hydrogen atom is in its second excited state? Justify your answer. Calculate the ratio of the maximum and minimum wavelengths of the radiations emitted in this process. [All India 2010C]

SOL: Bohr's postulates

1) Every atom consists of small and massive central core, known as nucleus around which electron revolve and necessary centripetal force prevailed by electrostatic force of attraction between positively charged nucleus and negatively charged electrons.

2) The electrons are revolved around the nucleus in only those circular orbits which satisfy the quantum condition that the angular momentum of electrons is equal to integral multiple of $\frac{h}{2\pi}$, where, h is

Planck's constant.

$$mvr = \frac{nh}{2\pi}$$

where, $n=1, 2, 3, ...$

In second excited state i.e., $n = 3$, two spectral lines namely Lyman series and Balmer series can be obtained corresponding to transition of electron from $n=3$ to $n=1$ and $n=3$ to $n=2$ respectively.

For Lyman series, $n = 3$ to $n = 1$, for minimum wavelength

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9}$$

For Balmer series (Maximum wavelength)

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= \left(\frac{9-4}{36}\right)R = \frac{5R}{36}$$

Divide Eq. (i) by Eq. (ii), we get

$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{\frac{8R}{9}}{\frac{5R}{36}} = \frac{8R}{9} \times \frac{36}{5R} = \frac{32}{5}$$

$$\lambda_{max} : \lambda_{min} = 32:5.$$

33) The energy of the electron, in the ground state of hydrogen, is –13.6 eV. Calculate the energy of the photon that would be emitted if the electron were to make a transition corresponding to the emission of the first line of the

a) Lyman series

b) Balmer series of the hydrogen spectrum. [All India 2009C]

SOL: Given, the energy of the electron, in the ground state of hydrogen is –13.6 eV.

$E_1 = -13.6$ eV

For $n = 2$, $E_2 = -3.4$ eV

For $n = 3$, $E_3 = -1.5$ eV $\left[\because E_n = -\frac{13.6}{n^2} \text{ eV}\right]$

Energy of photon corresponding to the first line of the

(a) Lyman series, $E = E_2 - E_1 = (-3.4) - (-13.6) = 10.2$ eV

(b) Balmer series, $E = E_3 - E_2 = (-1.5\text{eV}) - (-3.4) = 1.9$ eV.

34) The ground state energy of hydrogen atom is –13.6 eV.

a) What is kinetic energy of an electron in the 2nd excited state?

b) If the electron jumps to the ground state from the 2nd excited state, calculate the wavelength of the spectral line emitted. [All India 2008]

SOL: Ground state energy $E_i = -13.6$ eV

$$\because \quad E_n = -\frac{13.6}{n^2} \text{ eV}$$

(a) KE of an electron = – Total energy of electron

In second excited state. $n = 3$

Total energy $E_n = -\frac{13.6}{3^2} = -1.5$ eV

KE of the electron in 2nd excited state = $E_3 = -(-1.5 \text{ eV}) = 1.5$ eV.

(b) Transition of electron from $n_2 = 3$ to $n_1 = 1$

We have wavelength of emitted radiation

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= R\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda} = \frac{8R}{9}$$

$$\lambda = \frac{9}{8R} = \frac{9}{8 \times 1.09 \times 10^7}$$

$$\lambda = 1.03 \times 10^{-7} \text{m}$$

$$\lambda = 1030 \text{ Å}$$

The wavelength of emitted electron is 1030 $\overset{o}{\mathbf{A}}$.

35) The ground state energy of hydrogen atom is –13.6 eV. The photon emitted during the transition of electron from *n* = 3 to *n* =1 state, is incident on a photosensitive material of unknown work function. The photoelectrons are emitted from the materials with a maximum kinetic energy of 9eV. Calculate the threshold wavelength of the material used. [Foreign 2008]

SOL: Given, ground state energy E_1 = –13.6 eV

Energy of electron in nth orbit

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

For $n = 1$,

$$E_1 = -13.6 \text{ eV}$$

For $n = 3$,

$$E_3 = -\frac{13.6}{3^2} = -1.5 \text{ eV}$$

The energy of photon released during the transition of electron from n = 3 to n = l is

E = E_3 –E_1 = – (1.5) – (–13.6) =12.1 eV

Now, Einstein's photoelectric equation Energy of photon *(E)* = KE_{max} + W_o

where, W_o is work function of metal

12.1 eV = 9eV + W_o

W_o = 12.1 – 9 = 3.1 eV

Work function W_o= 3.1 eV

The wavelength correspond to 3.1 eV, threshold wavelength

$$\lambda_o = \frac{1242\ eV - nm}{W_o(\text{ineV})} = \frac{1242\ eV - nm}{3.1eV} = 401nm$$

36) (a) Using postulates of Bohr's theory of hydrogen atom, show that

i) **the radii of orbits increase as n^2 and**
ii) **the total energy of the electron increases as $1/n^2$, where *n* is the principal quantum number of the atom.**

(b) Calculate the wavelength of H_α line in Balmer series of hydrogen atom, given Rydberg constant $R = 1.097 \times 10^7\ m^{-1}$.

SOL: (b) For Balmer series α -line, wavelength is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where, $n = 3$ (for α-line)

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= \left(\frac{9-4}{36}\right)R$$

$$\frac{1}{\lambda} = \frac{5R}{36}$$

$$\lambda = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7}$$

$$\lambda = 6.563 \times 10^{-7}\ \text{m}$$

$$\lambda = 6563\ \text{Å}.$$

37) The total energy (potential + kinetic) of an electron in the ground state of Bohr model of hydrogen atom is –13.6 eV. Obtain the values of the potential energy *U* and kinetic energy *K* in eV. Include –ve or + ve sign as required. Would your answers change if the choice of the zero potential energy is changed ?

SOL:

The kinetic and the potential energies of electron in the hydrogen atom ($Z = 1$) are given by

$$K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} \quad \text{and} \quad U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r},$$

where r is the radius of the orbit in the given energy state. The total energy is

$$E = K + U = \frac{1}{4\pi\varepsilon_0}\left(\frac{e^2}{2r} - \frac{e^2}{r}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}.$$

Thus, $K = -E = -(-13.6\ \text{eV}) = +\mathbf{13.6\ eV}$

and $U = 2E = 2 \times (-13.6\ \text{eV}) = -\mathbf{27.2\ eV}.$

The above answers are based on the choice that the zero of the potential energy is at infinity. If a different zero of potential energy is chosen, then the potential energy and also the total energy would change, but the kinetic energy will still remain + 13.6 eV.

38) The ground state energy of hydrogen atom is –13.6 eV.

i) What is the potential energy and kinetic energy of an electron in the 2nd excited state ?

ii) If the electron jumps to the ground state from the 2nd excited state, calculate the wavelength of the photon emitted. (*h* = 6.626 × 10^{-34} Js)

SOL:

(i) The kinetic energy of an electron in the *n*th orbit of an atom is

$$K = \frac{Rhc}{n^2} = \frac{13.6}{n^2}\ \text{eV}.$$

∴ The kinetic energy of an electron in the 2nd excited state is ($n = 3$)

$$K = \frac{13.6\ \text{eV}}{(3)^2} = \mathbf{1.51\ eV.}$$

The potential energy of an electron in the 2nd excited state is

$$U = -\frac{2 \times 13.6}{(3)^2} = -\ \mathbf{3.02\ eV.}$$

(ii) Total energy of an electron in the 2nd excited state

$$E = K + U = 1.51\ \text{eV} + (-3.02\ \text{eV}) = -1.51\ \text{eV}$$

when the electron jumps to the ground state from the 2nd excited state, energy released is given by

$$\Delta E = -1.51\ \text{eV} - (-13.6\ \text{eV}) = 12.09\ \text{eV}.$$

∴ The corresponding wavelength of the emitted photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{6.626 \times 10^{-34}\ \text{Js} \times 3.0 \times 10^{8}\ \text{m s}^{-1}}{12.09\ \text{eV} \times 1.6 \times 10^{-19}\ \text{J/eV}} = 1.027 \times 10^{-7}\ \text{m} = \mathbf{1027\ \text{Å}}$$

39) The ground state energy of hydrogen atom is –13.6 eV

(i) What are the potential and kinetic energy of an electron in the 3rd excited state?
(ii) If the electron jumps to the ground state from the third excited state, calculate the frequency of photon emitted. [CBSE Sample Paper]

SOL: Given, the ground state energy of hydrogen atom is –13.6 eV.

$E_1 = -13.6$ eV

(a) Energy $E_n = \frac{-13.6}{n^2}$ eV

For third excited state,

$n = 4$

$$\therefore \quad E_4 = \frac{-13.6}{(4)^2}\ \text{eV} = \frac{-13.6}{16}\ \text{eV}$$

$$E_4 = -0.85\ \text{eV}$$

Kinetic energy KE = 0.85 eV and Potential energy = 2E = 2 × 0.85 eV = –1.7 eV

(b) Change in energy

$$\Delta E = E_4 - E_1$$
$$= [-0.85 - (-13.6)]\ \text{eV}$$
$$\Delta E = 12.75\ \text{eV}$$

$$\therefore \text{Frequency}\ \ \nu = \frac{\Delta E}{h}$$

$$= \frac{12.75 \text{ eV}}{6.63 \times 10^{-34}}$$

$$= \frac{12.75 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$\nu = 3 \times 10^{15} \text{ Hz}$$

40) The short wavelength limits of the Lyman, Paschen and Balmer series, in the hydrogen spectrum are denoted by λ_L, λ_P and λ_B respectively. Arrange these wavelengths in increasing order. [CBSE Sample Paper]

SOL:

Short wavelength $\lambda_{min} = \frac{n^2}{R}$

For Lyman series, $n = 1$

$\therefore \quad \lambda_L = \frac{1}{R}$

For Balmer series, $n = 2$

$\lambda_B = \frac{4}{R}$

For Paschen series

$\lambda_P = \frac{9}{R}$

Hence, $\lambda_L < \lambda_B < \lambda_P$

41) Calculate the ratio of energies of photons produced due to transition of electron of hydrogen atom from its,

***(i)* Second permitted energy level to the first level, and**
***(ii)* Highest permitted energy level to the second permitted level [S.P.]**

SOL: For transition of electron

(i) From second permitted energy level to first level

$E_1 = -3.4-(-13.6) = 10.2$ eV

(ii) From highest permitted energy level to the second level.

$E_2 = 0 - (-3.4) = 3.4$ eV

Thus $\frac{E_1}{E_2} = \frac{10.2}{3.4} = \frac{3}{1}$

42) The spectrum of a star in the visible and the ultraviolet region was observed and the wavelength of some of the lines that could be identified were found to be : 824 $\mathring{A}$, 970 $\mathring{A}$, 1120 $\mathring{A}$, 2504 $\mathring{A}$, 5173 $\mathring{A}$, 6100 $\mathring{A}$.
Which ofthese lines cannot belong to hydrogen atom spectrum? (Given Rydberg constant $R = 1.03 \times 10^7 m^{-1}$ and $\frac{1}{R} = 970 \mathring{A}$. Support your answer with suitable calculations. [S.P.]

SOL: `Lyman series' belongs to ultraviolet region and 'Balmer series' belongs to visible part of the spectrum. Minimum wavelength belonging to Lyman series is

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty}\right]$$

$$\lambda_{min} = \frac{1}{R} = 970 \text{ Å}$$

Similarly for other values of n, permitted values are 1293.3 Å, 1091 Å, 1034.6 Å, ... 970 Å

Maximum wavelength belonging to Balmer series is

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\lambda_{max} = \frac{36}{5}\left(\frac{1}{R}\right) = 6984 \text{ Å}$$

Similarly for other values of n, permitted values are 6984 Å, 5173.3 Å, 4619 Å, ….. 3880 Å

These values *i.e.* 824 Å, 1120 Å, 2504 Å, 6100 Å does not satisfy the given condition, so they cannot belong to hydrogen atom.

43) In a hydrogen atom, a transition takes place from *n* = 3 to n = 2 orbit. Calculate the wavelength of the emitted photon. Will the photon be visible ? To which spectral series will this photon belong ? (*R* = 1.097 x 10^7 m^{-1})

SOL: $n_1 = n_f$ and $n_2 = n_i$

The wavelength λ of emitted photon, when electron in an atom transists from a higher level n_2 to a lower level n_1, is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

When the transition takes place from $n = 3$ to $n = 2$, then for the wavelength of the emitted photon, we have

$$\frac{1}{\lambda} = 1{\cdot}097 \times 10^7 \text{ m}^{-1}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = (1{\cdot}097 \times 10^7 \text{ m}^{-1}) \times \frac{5}{36}.$$

$$\therefore \quad \lambda = \frac{36}{(1{\cdot}097 \times 10^7 \text{ m}^{-1}) \times 5} = 6{\cdot}563 \times 10^{-7} \text{ m} = \mathbf{6563 \text{ Å}}.$$

This wavelength falls in the visible (red) part of the spectrum, hence the photon will be visible. This photon is the first member of the Balmer series.

44) The wavelength of first member of the Lyman series is 1,216 Å. Calculate the wavelength of second member of the Balmer series.

SOL: The wavelengths of the different members of Lyman series are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right), \quad \text{where } n_i = 2, 3, 4, \ldots\ldots$$

Let λ_1 be the wavelength of first member of the Lyman series ($n_i = 2$). Then,

$$\frac{1}{\lambda_1} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R_H}{4} \quad ...(i)$$

The wavelengths of different members of the Balmer series are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right), \quad \text{where } n_i = 3, 4, 5, ...$$

Let λ_2' be the wavelength of the second member of Balmer series ($n_i = 4$). Then,

$$\frac{1}{\lambda_2'} = R_H\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R_H}{16} \quad ...(ii)$$

Dividing the equation (*i*) by (*ii*), we have

$$\frac{\lambda_2'}{\lambda_1} = \frac{3R_H}{4} \times \frac{16}{3R_H} = 4$$

or $\lambda_2' = 4\lambda_1$

Here, $\lambda_1 = 1,216$ Å

$\therefore$ $\lambda_2' = 4 \times 1,216 = \mathbf{4,864}$ **Å**

45) The energy of the electron in the ground state of hydrogen atom is - 13.6 eV.

- **i) What does the negative sign signify?**
- **ii) How much energy is required to take an electron in this atom from the ground state to the first excited state?**

SOL:

(i) It signifies that, the electron can be made free, from the nuclear attraction, if 13·6 eV energy is imparted to it, in the ground state.

(ii) Energy of an electron in n^{th} orbit of an H-atom

$$E_n = -\frac{13{\cdot}6}{n^2} \text{ eV}$$

Energy in ground state, *i.e.*, $n = 1$ state

$$E_1 = -13{\cdot}6 \text{ eV}$$

and in I^{st} excited state, *i.e.*, in, $n = 2$

$$E_2 = -\frac{13.{\cdot}6}{2^2} = -3{\cdot}4 \text{ eV}$$

. Energy required, to promote the electron, from, $n = 1$ to $n = 2$

$$\Delta E = E_2 - E_1 = -3{\cdot}4 - (-13{\cdot}6) = 10{\cdot}2 \text{ eV}$$

46) Using Bohr's formula for energy quantisation, find (i) the excitation energy of *a* = 3 level of He^+ ion and (ii) the ionisation potential of the ground state of Li^{++} ion.

SOL:

$$E = -Z^2 \frac{13{\cdot}6}{n^2} \text{ eV}. \text{ This is Bohr's formula.}$$

(i) For He^+ ion $(Z = 2)$, the excitation energy of $n = 3$ level is

$$E_3 - E_1 = -(2)^2 (13{\cdot}6 \text{ eV}) \left(\frac{1}{3^2} - \frac{1}{1^2}\right) = 48{\cdot}4 \text{ eV}.$$

(ii) The ionisation energy of Li^{++} $(Z = 3)$ is

$$E_\infty - E_1 = 0 - \left\{-(3)^2 \frac{13{\cdot}6 \text{ eV}}{12}\right\} = 122{\cdot}4 \text{ eV}.$$

Hence, the ionisation potential is 122·4 volt.

47) In the Rutherford scattering experiment the distance of closest approach for an α -particle is do. If α-particle is replaced by a proton, how much kinetic energy in comparison to α -particle will it require to have the same distance of closest approach do?

SOL:

For, an alpha particle, the distance of closest approach,

$$r_0 = \frac{1}{4\pi\varepsilon_0} \frac{Ze \times 2e}{K}$$

and for a proton,
$$r_0' = \frac{1}{4\pi\varepsilon_0} \frac{Ze \times e}{K'}$$

Now, $r_0 = r_0'$ (given)

$$\therefore \frac{1}{4\pi\varepsilon_0} \frac{Ze \times 2e}{K} = \frac{1}{4\pi\varepsilon_0} \frac{Ze \times e}{K'}$$

which gives,
$$K' = \frac{K}{2}$$

48) **If the number of particles scattered in the Rutherford experiment at 60^0 is 100 /min then find the number of particle scattered at 90^0.** [ANS: 25 per min]

Hint: Formula used $N \propto \dfrac{1}{\sin^4(\theta/2)}$.

49) **Calculate the wavelength of the second member of the Lyman series of hydrogen. *(R* = 1.1 ×10^7 m^{-1})**

Ans. 1023 $\overset{0}{A}$.

50) **Calculate the wavelengths of the first three lines of the Balmer series.**

[Ans. 6563 $\overset{0}{A}$, 4861 $\overset{0}{A}$, 4341 $\overset{0}{A}$.]

51) **Calculate the wavelengths of the Hα and the last lines of the Balmer sries.**

[Ans. 6563 $\overset{0}{A}$, 3646 $\overset{0}{A}$.]

52) **Calculate the wavelengths of the first lines of Balmer, Lyman and Paschen series.**

[Ans. 6563 $\overset{0}{A}$, 1215 $\overset{0}{A}$, 18752 $\overset{0}{A}$.]

53) **If the wavelength of the first line of Lyman series in the spectrum of hydrogen atom is 1215 Å what will be the wavelength of the second line in its Balmer series ?**[Ans. (27/5)λ]

54) **The wavelength of the first line of Balmer series is 6563 Å . Find the wavelength of the first line of the Lyman series.** [Ans. 1215 Å .]

55) **State the basic postulates of Bohr's atomic model and derive an expression for the radius of a stationary orbit. Prove that the various stationary orbits are not equally spaced.**

56) **Prove that the radius of the nth Bohr orbit of an atom is directly proportional to n^2, where *n* is the principal quantum number.**

57) **The value of ground state energy of hydrogen atom is –13.6 eV. (i) What does the negative sign signify ? *(ii)* How much energy is required to take an electron in this atom from the ground state to the second excited state.**

58) **Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels.[Delhi 2013]**

59) **Using Bohr's postulates for hydrogen atom, show that the total energy *(E)* of the electron in the stationary states can be expressed as the sum of kinetic energy *(K)* and potential energy *(U)*, where $K = -2U$. Hence deduce the expression for the total energy in the nth energy level of hydrogen atom. [Foreign 2012]**

60) **Using the postulates of Bohr's model of hydrogen atom, obtain an expression for the frequency of radiation emitted when the atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number $n_f (n_f < n_i)$. [Foreign 2011]**

61) **Draw a schematic arrangement of the Geiger-Marsden experiment for studying α -particle scattering by a thin foil of gold. Describe briefly, by drawing trajectories of the scattered α -particles. How this study can be used to estimate the size of the nucleus?[Foreign 2010]**

62) **Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels. [Delhi 2013]**

RADIOACTIVITY Pg No 97

PRACTICE QUESTIONS (STRUCTURE OF NUCLEUS)

1) Protons and neutrons exist together in an extremely small space within the nucleus. How is this possible when protons repel each other?

SOL: The very strong, attractive, short-range and charge-independent nuclear forces hold the nucleons together in the nucleus.

2) Write any two characteristic properties of nuclear force. [All India 2011]

SOL: Two characteristics of nuclear force

(i)These are short range force.

(ii)These are strong force of attractive nature.

(iii) independent of charge.

3) Write two characteristic features of nuclear force which distinguish it from the coulomb force.

SOL: Much stronger and charge independent.

4) You are given two nuclei ${}_3X^7$ and ${}_3Y^4$ (i) Are they isotopes of the same element? (ii) State with reason, which one of the two nuclei is likely to be more stable.

SOL: (i) Yes, the two nuclei are the isotopes of the same element.

(ii) ${}_3X^7$ has 3 protons and 4 neutrons; while ${}_3Y^4$ has 3 protons and 1 neutron. So ${}_3X^7$ experiences less proton-proton electrostatic repulsion compared to nuclear-force attraction than ${}_3Y^4$. Hence ${}_3X^7$ is more stable than ${}_3Y^4$.

5) Give the mass number and atomic number of elements on the right-hand side of the decay process : ${}_{86}Rn^{220} \rightarrow Po + He$

ANS: Mass number and atomic number of Po are 216, 84 respectively (${}_{84}Po^{216}$), and of He are 4 and 2 respectively (${}_2He^4$) .

6) If both the numbers of protons and neutrons are conserved in a nuclear reaction like

$${}_6C^{12} + {}_6C^{12} \longrightarrow {}_{10}N^{20} + {}_2He^4$$

In what way is mass converted into energy? Explain. [Delhi 2010]

SOL: The sum of masses of nuclei of product element is less than sum of masses of reactants and hence loss of mass take place during the reaction. This difference of mass of product element and reactant converts into energy and liberating in the form of heat.

Here, sum of masses of ${}_{10}Ne^{20}$ and ${}_{2}He^{4}$ is less than the sum of 2 ${}_{6}C^{12}$ and convension of this mass defect used to produce energy.

7) The mass of a nucleus in its ground state is always less than the total mass of its constituents neutrons and protons. Explain. [All India 2009]

SOL: a) Mass defect occurs in nucleus which converts into energy as per Einstein is mass energy relation $E = MC^2$ and produces binding energy. This energy binds nucleus together due to nuclear forces inspite of repulsive Coulombian forces.

8) Calculate the energy released in the following nuclear reaction

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + Q$$

[Mass of ${}^{238}_{92}U$ = 238.05079 u, Mass of ${}^{234}_{90}Th$ = 234.043630 u, Mass of ${}^{4}_{2}He$ = 4.002600 u, 1u = 931.5 MeV] [All India 2008][2007]

Is this decay spontaneous ? Give reason. ***(CBSE 2008, 07)***

SOL: Sum of masses of ${}^{234}_{90}Th$ and ${}^{4}_{2}He$ = 234:043630 + 4.002600 = 238.046230 u

Mass of ${}^{238}_{92}U$ = 238.05078

Loss of mass (mass defect) in given nuclear reaction

= 238.05078 – 238.046230 = 0.00455 u

Energy released in nuclear reaction = 0.00455 × 931.5 MeV = 4.238325 MeV

Yes above decay is spontaneous, as the nucleous has the tendency to convert to more stable nucleous. ${}_{92}U^{238}$ nucleus is unstable against α-decay.

9) Sketch a graph showing the variation of binding energy per nucleon as a function of mass number A, for large number of nuclei. State briefly, from which region of the graph, can release of energy in the process of nuclear fusion be explained. [Foreign 2008]

SOL: in the range of mass number 2 to 20, these are maximum and minimum as the curve where He, ${}_{6}C^{12}$ and ${}_{8}O^{16}$ are at maxima and ${}_{1}H^{2}$, Li,N etc at minima. This range of mass number may facilitate release of energy in nuclear fusion e.g., two ${}_{1}H^{2}$ nuclei of low Binding energy when combine in nuclear fusion to form ${}_{2}He^{4}$ of high binding energy per nucleon. In this process, energy will release in the form of heat.

10) **Define the Q-value of a nuclear process. When can a nuclear process not proceed spontaneously? It both the number of protons and the number of neutrons are conserved in a nuclear reaction, in what way is mass converted into energy (or *vice-versa)* in a nuclear reaction? [All India 2010C]**

SOL: The Q-value of a nuclear process refers the energy release in the nuclear process which can be determined using Einstein's mass energy relation $E = mc^2$. The Q-value is equal to the difference of mass of products and reactance multiplied by square of velocity of length.
The nuclear process does not proceed spontaneously when Q-value of a process is negative or sum of masses of product in greater than sum of masses of reactant.

11) **Draw the graph to show variation of binding energy per nucleon with mass number of different atomic nuclei. Calculate binding energy/nucleon of $_{20}Ca^{40}$ nucleus. Given, mass of $_{20}Ca^{40}$ = 39.962589 u; mass of proton = 1.007825 u; mass of neutron = 1.008665 u and 1u = 931MeV/c^2 [All India 2007]**

SOL:

Mass of $_{20}Ca^{40}$ nucleus : 39.962589 u
Sum of masses of nucleons of

$$_{20}Ca^{40} = 20\, m_p + 20\, m_n$$
$$= 20 \times 1.007825 + 20 \times 1.008665$$
$$= 40.329800 \text{ u}$$

Mass defect = Sum of masses of nucleon –Mass of nucleus = 40.3298 –39.962589 = 0.367211amu
Binding energy = 0.367211×931 MeV = 341.87344 MeV
Binding energy per nucleon = 8.546836 MeV/Nuclei

12) **Why is the mass of a nucleus always less than the sum of the masses of its constituents, neutrons and protons? If the total number of neutrons and protons in a nuclear reaction is conserved, how then is the energy absorbed or evolved in the reaction? Explain.[Delhi 2006]**

SOL: This difference of mass of sum of masses of nucleons and mass of nucleus is known as mass defect which converts into energy as per
Einstein's mass energy relationship $E = mc^2$.
This energy is known as binding energy which is used to hold the nucleus together inspite of Repulsive Coulombian force between positively charged protons. Otherwise nucleons will no longer be stable. The number of neutrons and protons is conserved, but it does not mean that the parent nuclei and product nuclei are same. The difference in the mass of parent and product nuclei converts into energy.

13) Write three characteristic properties of nuclear force which distinguish it from the electrostatic force. [All India 2006C]

SOL:

	Nuclear force	**Electrostatic force.**
1	Strongest short range force which operate upto distance of 2-3 fm.	It is not very short range force necessarily.
2	It does not obey inverse square law	It obey inverse square law
3	It exhibit charge independent character.	It depends on the nature of charge ,like charge repel where as opposite charge attracts each other.

14) The nucleus of an atom of $^{235}_{92}Y$, initially at rest, decays by emitting an α -particle as per the equation.

$$^{235}_{92}Y \rightarrow ^{231}_{90}X + ^{4}_{2}He + Energy$$

It is given that the binding energies per nucleon of the parent and the daughter nuclei are 7.8 MeV and 7.835 MeV respectively and that of α -particle is 7.07MeV/nucleon. Assuming the daughter nucleus to be formed in the unexcited state and neglecting its share in the energy of the reaction, calculate the speed of the emitted -particle. Take mass of α -particle to be 6.68 × 10 $^{-27}$ kg. [S.P.]

SOL: Total B.E. of parent nuclei = 235 × 7.8 MeV = 1833 MeV

Total B.F. of daughter nuclei = 231 × 7.835 = 1809.9 MeV

Total B.E. of α -Particle = 4 × 7.07 = 28.3 MeV

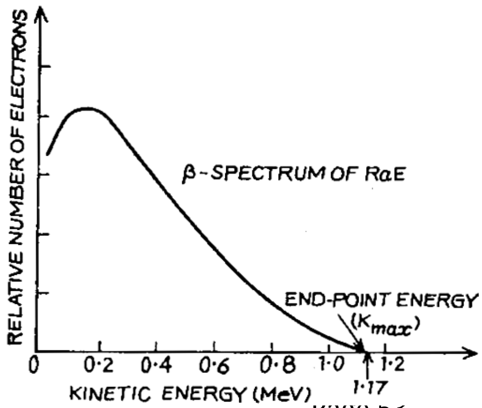
K.E. of α - particle = (1809.9 + 28.3) –1833 = 5.2 MeV

$$\frac{1}{2}mv^2 = 5.2\,\text{MeV}$$

$$v^2 = \frac{2\times5.2\times1.6\times10^{-19}}{6.68\times10^{-27}} = 2.5\times10^{14}$$

$$v = 1.58\times10^7\, m/s$$

15) Plot the distribution of kinetic energy of β particles and state why the energy spectrum is continuous.



SOL: The β particles emitted from a given radioactive nucleus have a continuous distribution of kinetic energies from zero to a maximum value K_{max} , the value K_{max} called the end point energy is characterstic of the emitter. It is 1.17 MeV for RaE ,as shown in graph

16) Calculate the energy in joule equivalent to the mass of one proton. The mass of proton is 1.00728 u. Express the energy in kilowatt-hour also.

SOL: The Einstein's energy – mass relation is

$$\Delta E = (\Delta m) c^2 .$$

Here $\Delta m = 1{\cdot}00728 \text{ u} = 1{\cdot}00728 \text{ u} \times (1{\cdot}66 \times 10^{-27} \text{ kg/u}) = 1{\cdot}672 \times 10^{-27} \text{ kg}.$

$$\therefore \ \Delta E = (1{\cdot}672 \times 10^{-27} \text{ kg})(3{\cdot}0 \times 10^8 \text{ m s}^{-1})^2 = \mathbf{1{\cdot}5 \times 10^{-10} \ J.}$$

In terms of kilowatt-hour, we can write

$$\Delta E = \frac{1{\cdot}5 \times 10^{-10} \text{ W s}}{(60 \times 60) \text{ s/h}} \qquad [\text{J} = \text{W s}$$

$$= 4{\cdot}2 \times 10^{-14} \text{ W h} = \mathbf{4{\cdot}2 \times 10^{-17} \ kW\,h} .$$

17) If the nucleons of a nucleus are separated far apart from each other, the sum of masses of all these nucleons is larger than the mass of the nucleus. Where does this mass difference come from? Calculate the energy released if ^{238}U nucleus emits an α -particle.
Given Atomic mass of ^{238}U = 238.0508 u
Atomic mass of ^{234}U = 234.04363 u
Atomic mass of α -particle = 4.00260 u and 1 u = 931 MeV [All India 2007]

SOL: The energy consumed in separating the nucleons for apart from each other in against of binding forces converts into the mass as per Einstein's mass energy relation $E = mc^2$ and hence mass difference occurs. α - decay of ^{238}U;

$$^{238}\text{U} \xrightarrow{\alpha} {}^{238}\text{U} + \alpha$$

The mass of parent nuclei ^{238}U, m_1 = 238.0508 u
The mass of product nuclei m_2 = mass of 234U ± mass of α particle
= 234.04363 + 4.00260 = 238.04623
Mass defect $m = m_1 - m_2$ =238.0508 –238.04623 =0.00457
Energy released =0.00457× 931meV = 425 meV.

18) A nucleus $_{10}Ne^{23}$ undergoes $\overline{\beta}$ decay and becomes $_{11}Na^{23}$. Calculate the maximum kinetic energy of electrons emitted assuming that the daughter nucleus and anti-neutrino carry negligible kinetic energy. *Mass* of $_{10}Ne^{23}$ = 22.994466 u , mass of $_{11}Na^{23}$ = 22.989770u , 1u = 931.5 MeV/c^2.
SOL:

The equation representing $_{-1}\beta^0$ decay of $_{10}Ne^{23}$ is

$$_{10}Ne^{23} \longrightarrow {}_{11}Na^{23} + {}_{-1}\beta^0 + \bar{\nu} + Q$$

where Q is kinetic energy shared by $_{10}Ne^{23}$ and $_{11}Na^{23}$.

Ignoring the rest mass of $_{-1}\beta^0$ and anti-neutrino $(\bar{\nu})$.

$$\text{Mass defect } \Delta m = \text{mass of Ne} - \text{mass of Na}$$
$$= 22{\cdot}994466 - 22{\cdot}989770 = 0{\cdot}004696 \text{ u}$$

$\therefore\ Q = \Delta m c^2\ (0{\cdot}004696) \times 931{\cdot}5\ \text{MeV/c}^2 \times c^2 = 4{\cdot}374\ \text{MeV}$

$\therefore$ maximum kinetic energy of $_{-1}\beta^0$ = **4·374 MeV.**

19) If nuclei with lower binding energy per nucleon transform to nuclei with greater binding energy per nucleon, would the reaction be exothermic or endothermic? Justify your answer and write two examples to support your answer. [CBSE Sample Paper]

SOL: When a nucleus with less binding energy per nucleon, transforms to a nuclei with greater binding energy per nucleon, a sufficient amount of energy will release. Hence, it is an exothermic reaction. **Examples** In case of **fission,** heavy nucleus decays into two or more lighter nuclei.
In case of **fusion**, two or more light nuclei fuse together to form a heavier nuclei.
In both the above processes, energy is released.

20) A neutron breaks into a proton ($_1H^1$) , an electron (β -particle) and antineutrino ($_0n^1 \rightarrow {}_1H^1 + {}_{-1}\beta^0 + \bar{\nu}$). Calculate the energy released in this process in MeV. Given mass of electron = 9 × 10^{-31} kg, mass of proton = 1.6725 × 10^{-27} kg, mass of neutron 1.6747 × 10^{-27} kg.

SOL:

$$\text{Mass of proton} = 1{\cdot}6725 \times 10^{-27}\ \text{kg.}$$
$$\text{Mass of electron} = 0{\cdot}0009 \times 10^{-27}\ \text{kg.}$$
$$\text{Mass of proton + electron} = 1{\cdot}6734 \times 10^{-27}\ \text{kg.}$$
$$\text{Mass of neutron} = 1{\cdot}6747 \times 10^{-27}\ \text{kg.}$$
$$\text{Their difference, } \Delta m = 0{\cdot}0013 \times 10^{-27}\ \text{kg.}$$

According to energy-mass relation, the energy released is

$$\Delta E = (\Delta m)\,c^2 = (0{\cdot}0013 \times 10^{-27}\ \text{kg}) \times (3{\cdot}0 \times 10^8\ \text{m s}^{-1})^2$$
$$= 1{\cdot}17 \times 10^{-13}\ \text{J} = \frac{1{\cdot}17 \times 10^{-13}\ \text{J}}{1{\cdot}6 \times 10^{-13}\ \text{J/MeV}} = \mathbf{0{\cdot}731\ MeV.}$$

21) Calculate the binding energy of an α -particle in MeV. The masses of proton, neutron and α -particle are 1.00728 , 1.00867 and 4.00151 u respectively.

SOL An α -particle is helium ($_2He^4$) nucleus, containing two protons and two neutrons. We can find out its mass defect from the given data :

$$\text{mass of 2 protons} = 2 \times 1{\cdot}00728 = 2{\cdot}01456\ \text{u}$$
$$\text{mass of 2 neutrons} = 2 \times 1{\cdot}00867 = 2{\cdot}01734\ \text{u}$$
$$\text{Total} = 4{\cdot}03190\ \text{u.}$$

This is the total mass of the 4 nucleons of α -particle. The mass of α -particle is 4.00151 u. Therefore, the mass defect is

$$\Delta m = \text{mass of nucleons} - \text{mass of } \alpha\text{-particle}$$
$$= 4.03190 \text{ u} - 4.00151 \text{ u} = 0.03039 \text{ u.}$$

According to the energy-mass relation $\Delta E = (\Delta m)\, c^2$, 1 u of mass is equivalent to 931·5 MeV of energy.

$\therefore$ energy equivalent to 0·03039 u is

$$\Delta E = 0.03039 \text{ u} \times 931.5 \text{ MeV/u} = \mathbf{28.3 \text{ MeV}}.$$

This is the binding energy of α -particle. The binding energy per nucleon is 28.3/4 = 7.07 MeV. This *large* amount of energy explains the high stability of α -particle.

22) Calculate the binding energy per nucleon of carbon ($_6C^{12}$) nucleus. Given : mass of carbon atom = 12.0000(1 u, mass of proton = 1.00728 u, mass of neutron = 1.00867 u, mass of electron =0.00055 u. The energy equivalent of 1 u is 931.5 MeV.

SOL: The carbon nucleus has 6 protons and 6 neutrons, and 6 orbital electrons revolving around the nucleus.

$$\text{Mass of 6 protons} = 6 \times 1.00728 = 6.04368 \text{ u}.$$
$$\text{Mass of 6 neutrons} = 6 \times 1.00867 = 6.05202 \text{ u}.$$
$$\text{Total} = 12.09570 \text{ u.}$$

This is the sum of the masses of the nucleons.

$$\text{Mass of carbon } ({}_6C^{12}) \textit{ atom} = 12.00000 \text{ u.}$$
$$\text{Mass of 6 (orbital) electrons} = 6 \times 0.00055 = 0.00330 \text{ u}.$$
$$\text{Difference} = 11.99670 \text{ u.}$$

This is the mass of the carbon *nucleus*. Therefore, the mass-defect is

$$\Delta m = \text{mass of nucleons} - \text{mass of nucleus} = 12.09570 \text{ u} - 11.99670 \text{ u} = 0.09900 \text{ u.}$$

Its energy equivalent is $0.09900 \text{ u} \times 931.5 \text{ MeV/u} = 92.22 \text{ MeV}$.

$$\therefore \text{binding energy per nucleon} = \frac{92.22 \text{ MeV}}{12} = \mathbf{7.68 \text{ MeV}}.$$

23) A neutron is absorbed by a $_3Li^6$ nucleus with subsequent emission of an α-particle. Write the corresponding nuclear reaction and calculate the energy released in the reaction.

Given : *m* ($_0n^1$) =1.008665 u , m ($_3Li^6$) = 6.015126 u , *m* ($_2He^5$ = 4.002603 u, m($_1H^3$) and

1 u × c^2 = 931.5 MeV.

SOL:

The nuclear reaction is

$${}_3Li^6 + {}_0n^1 \longrightarrow {}_1H^3 + {}_2He^4 \quad (\alpha\text{-particle}).$$

$$\text{Mass of } {}_3Li^6 + {}_0n^1 = 6.015126 \text{ u} + 1.008665 \text{ u} = 7.023791 \text{ u.}$$

$$\text{Mass of } {}_1H^3 + {}_2He^4 = 3.016049 \text{ u} + 4.002603 \text{ u} = 7.018652 \text{ u.}$$

$$\therefore \quad \text{mass loss, } \Delta m = 7.023791 \text{ u} - 7.018652 \text{ u} = 0.005139 \text{ u.}$$

Its energy equivalent is

$$\Delta E = (\Delta m) \times c^2 = (0.005139 \text{ u}) \times c^2.$$

Now, 1 u × c^2 = 931·5 MeV.

$$\therefore \quad \Delta E = 0.005139 \times 931.5 \text{ MeV} = \mathbf{4.787 \text{ MeV}}.$$

24) **Calculate the binding energy per nucleon of the nuclei of (i)$_{26}Fe^{56}$ and (ii) $_{83}Bi^{209}$. Given: mass of hydrogen atom = 1.007825 u , mass of neutron = 1008665 u , mass of 26Fe^{56} atom = 55.934939 u , mass of $_{83}Bi^{209}$ atom = 208.980388 u and 1 u × e^2 = 931.5 MeV. Which nucleus is more stable?**

SOL:

The *nuclear* binding energy of an atom $_ZX^A$ in terms of *atomic* masses and neutron mass is given by

$$BE = [Z\,m_H + (A-Z)\,m_n - m\,(_ZX^A)]\,c^2,$$

where m_H is the mass of hydrogen atom, m_n the mass of neutron and $m\,(_ZX^A)$ the mass of the *atom* $_ZX^A$, all the masses being in unified atomic mass unit u.

(i) For $_{26}Fe^{56}$: $Z = 26$, $A = 56$ and so $(A - Z) = 56 - 26 = 30$.

$$\therefore \quad BE = [(26 \times 1{\cdot}007825) + (30 \times 1{\cdot}008665) - 55{\cdot}934939]\text{ u} \times 931{\cdot}5\text{ MeV/u}$$
$$= [26{\cdot}20345 + 30{\cdot}25995 - 55{\cdot}934939]\text{ u} \times 931{\cdot}5\text{ MeV/u}$$
$$= 0{\cdot}528461\text{ u} \times 931{\cdot}5\text{ MeV/u} = 492{\cdot}26\text{ MeV}.$$

The nucleus of $_{26}Fe^{56}$ contains 56 nucleons. Therefore, its binding energy per nucleon is

$$\frac{492{\cdot}26}{56} = \mathbf{8{\cdot}790\ MeV.}$$

(ii) For $_{83}Bi^{209}$: $Z = 83$, $A = 209$ and so $(A - Z) = 209 - 83 = 126$.

$$\therefore \quad BE = [(83 \times 1{\cdot}007825) + (126 \times 1{\cdot}008665) - 208{\cdot}980388]\text{ u} \times 931{\cdot}5\text{ MeV/u}$$
$$= [83{\cdot}649475 + 127{\cdot}091790 - 208{\cdot}980388]\text{ u} \times 931{\cdot}5\text{ MeV/u}$$
$$= 1{\cdot}760877\text{ u} \times 931{\cdot}5\text{ MeV/u} = 1640{\cdot}26\text{ MeV}.$$

The nucleus of $_{83}Bi^{209}$ contains 209 nucleons. Therefore, its binding energy per nucleon is

$$\frac{1640{\cdot}26}{209} = \mathbf{7{\cdot}848\ MeV}.$$

Obviously, $_{26}Fe^{56}$ is more stable than $_{83}Bi^{209}$.

25) **The binding energies of deuteron ($_1H^2$) and α -particle ($_2He^4$) are 1.112 and 7.07 MeV/nucleon respectively. Find out : (i) which nucleus is more stable ? (ii) In the process, $_1H^2 + {_1H^2} \rightarrow {_2He^4}$, state whether energy is supplied or liberated and how much ?**

SOL:

(i) Higher the binding energy per nucleon for a nucleus, more stable is the nucleus. So, **α -particle ($_2He^4$) is more stable.**

(ii) Let us consider the nuclear reaction :

$$_1H^2 + {_1H^2} \longrightarrow {_2He^4}.$$

Deuteron ($_1H^2$) has 2 nucleons. So, binding-energy of deuteron = $2 \times 1{\cdot}112$ MeV = $2{\cdot}224$ MeV.

$\therefore$ total binding energy of $(_1H^2 + {_1H^2}) = 2{\cdot}224\text{ MeV} + 2{\cdot}224\text{ MeV} = 4{\cdot}448\text{ MeV}$.

$_2He^4$ has 4 nucleons. So, binding energy of $_2He^4$ is

$$4\text{ nucleons} \times 7{\cdot}07\text{ MeV/nucleon} = 28{\cdot}28\text{ MeV}.$$

Thus, $_2He^4$, in comparison to $(_1H^2 + {_1H^2})$, has $28{\cdot}28\text{ MeV} - 4{\cdot}448\text{ MeV} \simeq 23{\cdot}8\text{ MeV}$ more binding energy. So, **23·8 MeV energy will be liberated** in this process.

26) **When a deuteron ($_1H^2$) of mass 2.0141 u and negligible kinetic energy is absorbed by a lithium ($_3Li^6$) nucleus of mass 6.0155 u , the compound nucleus disintegrates spontaneously into two alpha particles, each of mass 4.0026 u . Calculate the energy in joule carried by each alpha particle. (1 u = 1.66 × 10 kg).**

SOL: The equation of nuclear disintegration is

$$_3Li^6 + {_1H^2} \longrightarrow {_2He^4} + {_2He^4} + Q.$$

The liberated energy Q is given by

$$Q = \Delta m \times c^2 = [m(_3Li^6) + m(_1H^2) - 2m(_2He^4)]c^2$$
$$= [6{\cdot}0155\text{ u} + 2{\cdot}0141\text{ u} - 2(4{\cdot}0026\text{ u})]c^2$$
$$= (0{\cdot}0244\text{ u})c^2.$$

Now, $1\text{ u} = 1{\cdot}66 \times 10^{-27}$ kg.

$$\therefore\; Q = [0{\cdot}0244\text{ u} \times (1{\cdot}66 \times 10^{-27}\text{ kg/u})]c^2$$
$$= [0{\cdot}0405 \times 10^{-27}\text{ kg}](3{\cdot}0 \times 10^8\text{ m s}^{-1})^2$$
$$= 3{\cdot}645 \times 10^{-12}\text{ J}.$$

This energy is equally shared by the two alpha particles. Hence energy carried by each alpha particle is **$1{\cdot}823 \times 10^{-12}$ J .**

27) **What is the meaning of atomic mass unit *(u)* ? Give its S.I. equivalent. Also calculate the energy released in MeV when 1 u matter disappear?**

Hint: 1 *u* = 1.660565× 10^{27} kg and 1 *u* F= 931.5 MeV .

28) **Calculate the binding energy per nucleon of $_{17}Cl^{35}$. Given : mass of $_{17}Cl^{35}$ = 34.980000 u ,mass of proton = 1.007825 u , mass of neutron = 1.008665 u and 1 u = 931.5 MeV.**

ANS: 8.22 MeV.

RADIOACTIVITY (NUMERICAL) Pg No 116

1) **Out of the two characteristics. The mass number (A) and the atomic number *(Z)* of a nucleus, which one does not change during nuclear β -decay? [All India 2008C]**

SOL: The mass number A of a nucleus does not change during nuclear β -decay.

2) **How is the mean life of a radioactive sample related to its half life?, [Foreign 2011]**

SOL: Mean life $\tau = \frac{1}{\lambda}$ where $\lambda =$ decay constant.

$$T = \frac{\log_e 2}{\lambda} = \frac{2.303 \log_{10} 2}{\lambda} = \frac{2.303 \times 0.3010}{\lambda} = \frac{0.693}{\lambda}$$

$$T = \tau \times 0.693 \qquad \left[\because \frac{1}{\lambda} = \tau = \frac{1}{\lambda}\right]$$

$$\tau = \frac{T}{0.693} = 1.44\ T$$

3) **A nucleus undergoes β^- -decay . How does its**
   a) **mass number(A)**
   b) **atomic number (Z) change? [Delhi 2011C]**

SOL: During β -decay
Since $\beta = {}_{-1}\beta^0$, therefore **(a)**No change in mass number. **(b)** Atomic number increases by 1.

4) **With the help of an example, explain how the neutron to proton ratio changes during alpha decay of a nucleus.**

SOL:

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}.$$

The initial neutron-proton ratio is $\frac{A-Z}{Z}$, and the final ratio is $\frac{(A-Z)-2}{Z-2}$. The ratio **increases slightly.**

5) **In a series of radioactive disintegration of $_zX^A$, first an α-particle and then. a β-particle is emitted. What are atomic number and mass number of the new nucleus formed ?**

ANS: Atomic number is Z –1 , mass number is A– 4

6) **When a radioactive nucleus $_zX^A$ emits the followings, how shall we write the residual nucleus in each case ? (i) An α -particle, (ii) a β-particle, (iii) a γ -photon**

ANS: (i) $_{z-2}Y^{A-4}$ (ii) $_{z+1}Y^A$ (iii) $_zX^A$

7) **Explain what is meant by radioactive decay ?**

SOL: Radioactive decay is the spontaneous disintegration of the nucleus of a radioactive atom with emission of α- or β- particles and γ-rays. It results into an atom of some other element.

8) Write the equations for the two types of β -decay. Why is it very difficult to detect the neutrino? [CBSE Sample Paper]

SOL:

In β^--decay

$${}_Z^A X \longrightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \bar{\nu}$$

In β^+-decay ${}_Z^A X \longrightarrow {}_{Z-1}^{A} Y + {}_{+1}^{0} e + \nu$

Neutrino is an uncharged particle which interacts very weakly with matter and hence, escapes undeflected.

9) What is the basic mechanism for the emission of β^- and β^+ particles in a nuclide? Give an example by writing explicitly a decay process for β^-emission. Is

a) the energy of the emitted β -particles continuous or discreate?

b) the daughter nucleus obtained through β -decay, an isotope or an isobar of the parent nucleus? [Delhi 2010C]

SOL: During β^--decay from the nucleus, nuclei undergoes a change in such a way that atomic number increases by one and mass number remain same.

In β^+-decay, the mass number of present radioactive nuclei remains same whereas atomic number decrease by one.

Example of β^--decay

$${}_{15}^{32}P \rightarrow {}_{16}^{32}S + e^- + \bar{\nu}$$ (Half life = 26 days)

In β -decay, an electron and an antineutrino are created in following manner :

$$n \rightarrow p + e^- + \bar{\nu}$$

(a) The energy of emitted β -particles is continuous.

(b) As there is no change in mass number during β -decay. So, the daughter nucleus is isobar of the parent nucleus.

10) Explain with an example, whether the neutron-proton ratio in a nucleus increases or decreases due to a beta (β) decay.

SOL:

$${}_Z X^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}\beta^0 + \bar{\nu}.$$

The initial neutron-proton ratio is $\frac{A-Z}{Z}$, and the final ratio is $\frac{A-(Z+1)}{Z+1}$. The ratio **decreases.**

11) Whether the neutron to proton ratio in a nucleus increase, decrease or remain same after the nucleus emits a position.

SOL:

$${}_Z X^A \longrightarrow {}_{Z-1}Y^A + {}_{+1}\beta^0 + \nu.$$

12) A radioactive nucleus A undergoes a series of decays according to the following scheme:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A_4 are 172 and 69 respectively. What are these numbers for A? [Delhi 2009]

SOL: In α -decay, the atomic number decreases by 2 units and mass number decreases by 4 units. In β -decay, the atomic number increases by 1 unit but mass number does not change. In γ -decay, there is no change in atomic number and mass number.
Let the mass number and atomic number of A be X and Y respectively. So,

$$ {}_{Y}A^{X} \xrightarrow{\alpha} {}_{y-2}A_1^{x-4} \xrightarrow{\beta} {}_{y-2+1}A_2^{x-4} $$

$$ \text{or} \quad {}_{y-1}A_2^{x-4} \xrightarrow{\alpha} {}_{y-1-2}A_3^{x-4-4} $$

$$ \text{or} \quad {}_{y-3}A_3^{x-8} \xrightarrow{\gamma} {}_{y-3}A_4^{x-8} $$

According to question, the mass number and atomic number of A_4 are 172 and 69.
x – 8 = 1 72
x = 1 72 + 8 = 180
y – 3 = 69 ; y = 72

13) A radioactive nucleus undergoes a series of decay according to the scheme :

$$ A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4 . $$

If the mass number and atomic number of A are 180 and 72 respectively, what are these numbers for A_4?

14) A radioactive nucleus A decays as $A \xrightarrow{\beta^+} A_1 \xrightarrow{\alpha} A_2$ If the mass number and atomic number of A_2 are 176 and 71 respectively, find these numbers for A_1 and A . Which of these are isobars?

SOL: For A_1 , 180 and 73; For A , 180 and 74. A and A_1 are isobars.

15) The radioactive isotope D decays according to the sequence

$$ D \xrightarrow{\beta\text{-Particle}} D_1 \xrightarrow{\alpha\text{-particle}} D_2 $$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what is
a) the mass number
b) atomic number of *D?* [Delhi 2007]

SOL: In β -decay, the mass number remains same and atomic number increases by 1 unit. In α-decay, the mass number decreases by 4 units and atomic number decreases by 2 units.
Let mass and atomic number of *D* be A and *Z* respectively.

$$ {}_{Z}D^{A} \xrightarrow{\beta-particle} {}_{Z+1}D_1^{A} \xrightarrow{\alpha-particle} {}_{Z+1-2}D_2^{A-4} \quad or \quad {}_{Z-1}D_2^{A-4} $$

According to question, the mass number and atomic number of D_2 are 176 and 71 respectively.
(a) Mass number of D_2= $Z-1$= 71
Hence Mass number of D is *Z* = 72
(b) Atomic number of D_2 = A – 4 = 176
Atomic number of D is A =176 + 4 =180

16) The sequence of stepwise decays of a radioactive nucleus is :

$$ D \xrightarrow{\alpha} D_1 \xrightarrow{\beta} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\alpha} D_4 $$

If the nucleon number and atomic number for D_2 are 176 and 71 respectively, what are the corresponding values for D and D_4 nuclei?

ANS: For D , 180, 72; and for D_4 ,168, 67.

17) The sequence of stepwise decay of a radioactive nucleus is

$$D \xrightarrow{-\alpha} D_1 \xrightarrow{-\beta} D_2$$

If the atomic number and mass number of D_2 are 71 and 176 respectively , what are their corresponding values for *D?* [Delhi 2010]

18) A nucleus $^{238}_{92}U$ undergoes alpha-decay and transforms to thorium. What is

a) the mass number and

b) atomic number of the nucleus produced? [Delhi 2011C]

SOL: In α -decay, the mass number of parent nucleus decreases by 4 units and atomic number decreases by 2 units.

$${}_{92}U^{238} \xrightarrow{-\alpha-particle} {}_{90}Th^{234}$$

(a) Mass number of produced nucleus = 234

Atomic number of produced nucleus =90

19) Define the term 'activity' of a radionuclide. Write its SI unit. [All India 2007]

SOL: The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.
SI unit of activity is Becquerel (Bq).
1 Bq = l disintegration/s.

20) Name the physical quantity whose SI unit is becquerel (Bq). How is this quantity related to (i) disintegration constant, (ii) half life and (iii) mean life of the radioactive element? [CBSE Sample Paper]

SOL: Becquerel (Bq) is the S I unit of activity.
(i) Relation between activity and disintegration constant
R = λ N ; where, λ is the decay constant.
(ii) Relation between activity and half life

$$R = \left(\frac{0.693}{T}\right)N$$

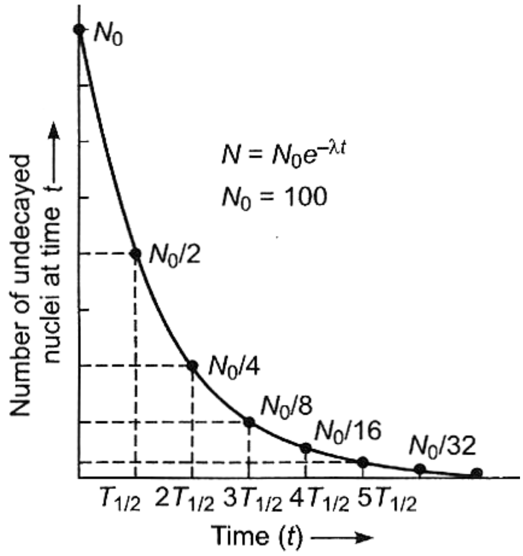
where, T is the half life.
(iii) Relation between activity and mean life of the radioactive element

$$R = \left(\frac{1}{\tau}\right)N$$

where, τ is the mean life.

21) Draw a plot representing the law of radioactive decay . Define the activity of a sample of a radioactive nucleus. Write its SI unit. [Foreign 2008]

SOL: The curve representing the law of radioactive decay is shown below.



Decay curve for a radioactive element

The rate of decay of a radioactive substance is called the 'activity' (R) of the substance, that is,

$$R = -\frac{dN}{dt}.$$

The **SI unit of activity is 'becquerel' (Bq) ;**

1 Bq = 1 disintegration s^{-1}

1curie (1c) = 3.7×10^{10} Bq

22) State the law of radioactive decay. Plot a graph showing the number (N) of undecayed nuclei as a function of time (t) for a given radioactive sample having half life $T_{1/2}$. Depict in the plot, the number of undecayed nuclei at (a) t = 3 $T_{1/2}$ and (b) t = 5 $T_{1/2}$. [Delhi 2011]

SOL: Law of radioactive decay The rate of disintegration of radioactive sample at any instant is directly proportional to the number of undisintegrated nuclei present in the sample at that instant *i.e.*,

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

where, N = number of undisintegrated nuclei present in the sample at any instant t and $\frac{dN}{dt}$ is rate of disintegration.

23) State the law of radioactive decay. Plot a graph showing the number (N) of undecayed nuclei as a function of time (t) for a given radioactive sample having half life $T_{1/2}$. Depict in the plot, the number of undecayed nuclei at (a) T = 2 $T_{1/2}$ and (b) T = 4 $T_{1/2}$ [Delhi 2011]

24) (a) Define 'activity' of a radioactive material and write its SI unit.

(b) Plot a graph showing variation of activity of a given radioactive sample with time.

25) A radionuclide sample has N_o nuclei at t = 0. Its number of undecayed nuclei get reduced to N = $\frac{N_o}{e}$ at t = τ. What does the term τ stand for? Write, in terms of τ, the time interval T in

which half of the original number of nuclei, of this radionuclide would have got decayed.[Delhi 2008C]

SOL: (i) The time τ in which number of undecayed nuclei get reduces to $\frac{N_o}{e}$ **is called** mean life or average life.

(ii) The time interval T in which half of original number of nuclei would have got decayed is $T = \frac{0.693}{\lambda} = 0.693\ \tau$.

26) (a) What is meant by half life of a radioactive element?
(b) the half life of a radioactive substance is 30 s. Calculate (i) the decay constant and (ii) time taken for the sample to decay by 3/4th of the initial value. [Foreign 2009]

SOL: Half life Half life of a radioactive element is the time taken by the sample to disintegrate to half of its original amount.

(ii) $\because \quad N = N_0\left(\frac{1}{2}\right)^n$

Half life period $T_{1/2} = \frac{0.693}{\lambda}$

where, λ is decay constant.

(b) $T_{1/2} = 30$ s

(i) $\lambda = ?$

$$\because \quad T_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{30} = 0.0231\,s^{-1}$$

where, n = number of half lives ; N= number of undisintegrated nuclei present in the sample.
N_o = original number of undisintegrated atom.

Here, $N = N_0 - \frac{3}{4}$

$$N_0 = \frac{1}{4}N_0$$

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$\frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n$$

$$n = 2$$

But number of half lives $= \frac{\text{Total time taken}}{\text{Half life}}$

$$2 = \frac{\text{Total time taken}}{(30\ s)}$$

Total time taken = 60 s = 1 min.

27) (a) What is meant by half-life of a radioactive element ?
(b) The half life of a radioactive substance is 20 s. Calculate (i) the decay constant and (ii) time taken for the sample to decay be 7/8th of the initial value. [Foreign 2009]

28) (a) What is meant by half life of a radioactive element?

(b) The half life of a radioactive substance is 50 s. Calculate (i) the decay constant and (ii) time taken for the sample to decay by 3/4th of the initial value. [Foreign 2009]

29) An observer , in a laboratory, starts with N_o nuclei of a radioactive sample and keeps on observing the number *(N)* of left over nuclei at regular intervals of 10 min each. She prepares the following table on the basis of her observation : Use this data to plot a graph of $\log_e(N_o/N)$ *vs* time (t) and calculate the

Time *t* (in min)	$\log_e\left(\frac{N_0}{N}\right)$
0	0
10	3.465
20	6.930
30	10.395
40	13.860

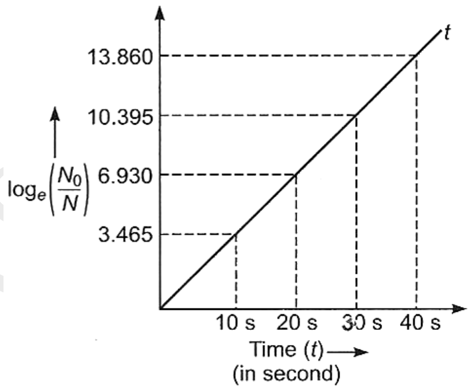
(a) decay constant and (b) half life of the given sample. [Delhi 2009C]

SOL:

(a) $\because \quad \log_e\left(\frac{N_0}{N}\right) = \lambda t$

$\Rightarrow$ Slope of $\log_e\left(\frac{N_0}{N}\right)$ *vs* time *t* graph gives decay constant (λ).

$\therefore \quad \lambda = \frac{3.465}{10} = \frac{6.930}{20} = 0.3465\ s^{-}$



(b) $\because$ Half life $T_{1/2} = \frac{0.693}{\lambda}$

$T_{1/2} = \frac{0.693}{0.3465} = 2s$

Half life $T_{1/2} = 2s$

30) The activity of a radioactive element reduces to 1/16 th of its original value in 30 years. Find the half-life and the decay constant of the element. [CBSE Sample Paper]

SOL: If the initial number of atoms in a radioactive material be N_0 , then the number of atom N remaining after n half- lives is given by

$$N = N_0\left(\frac{1}{2}\right)^n.$$

$$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n.$$

The activity is proportional to the number of atoms ($R = \lambda N$). Therefore, the number of remaining atoms is 1/16 of its initial value, that is, $\frac{N}{N_0} = \frac{1}{16}$.

$$\therefore \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n$$

or
$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^4$$

$\therefore \quad n = 4.$

The half-life T of the element is given by

$$T = \frac{\text{time taken in decay}}{\text{number of half-lives}} = \frac{30 \text{ years}}{4} = \mathbf{7{\cdot}5 \text{ years}}.$$

The decay constant of the element is given by

$$\lambda = \frac{\log_e 2}{T} = \frac{0{\cdot}6931}{7{\cdot}5 \text{ years}} = \mathbf{0{\cdot}09241 \text{ per year}}.$$

31) A radioactive material is reduced to $\frac{1}{16}$ of its original amount in 4 days. How much material should one begin with so that 4×10^{-3} kg of the material is left after 6 days. [S.P.]

SOL:

(*a*) Number of atoms left after '*n*' half-lives is $N = N_0\left(\frac{1}{2}\right)^n$

Where $n = \frac{t\,(\text{time of disintegration})}{T\,(\text{half life period})}$

$$\frac{N}{N_0} = \frac{1}{16} \text{ (given)} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^n$$

$$n = 4$$

$$\therefore \quad T = \frac{t}{4} = \frac{4}{4} = 1 \text{ day}$$

(*b*) It 4×10^{-3} kg of the material is left after 6 days, amount initially present is

$$N_0 = N\,(2)^6$$
$$= 4 \times 10^{-3} \times 64 = 256 \times 10^{-3} \text{ kg}$$

32) Two radioactive nuclei X and Y initially contain equal number of atoms. Their half-lives are1hour and 2 hours respectively. Calculate the ratio of their rates of disintegration after 2 hours.

SOL:

Let both X and Y have N_0 atoms initially. No. of atoms of X left undecayed after 2 hours (2 half-lives) is

$$N_X = N_0\left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

and that of Y after 2 hours (1 half-life) is

$$N_Y = N_0\left(\frac{1}{2}\right)^1 = \frac{N_0}{2}.$$

By Rutherford-Soddy law, the rate of disintegration is given by

$$-\frac{dN}{dt} = \lambda N$$

$$\therefore \quad \frac{-(dN_X/dt)}{-(dN_Y/dt)} = \frac{\lambda_X N_X}{\lambda_Y N_Y} = \frac{T_Y \frac{N_0}{4}}{T_X \frac{N_0}{2}} = \frac{2}{1} \times \frac{1}{2} = \mathbf{1}.$$

33) The half life of $_{92}U^{238}$ against α-decay is 4.5×10^9 years. Calculate the activity of 1 g sample of $_{92}U^{238}$. Avogadro's number is 6.023×10^{23}mole^{-1}.

SOL:

1 gram-atom of U^{238} has a mass of 238 g and contains $6{\cdot}023 \times 10^{23}$ atoms. Therefore, th

number of atoms in 1 g of U^{238} is

$$N = \frac{6.023 \times 10^{23}}{238} = 2.53 \times 10^{21}.$$

The half-life T of U^{238} is 4.5×10^9 years $(= 4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60 = 1.42 \times 10^{17}$ s)
Therefore, the decay constant of U^{238} is

$$\lambda = \frac{0.6931}{T} = \frac{0.6931}{1.42 \times 10^{17}\ s} = 0.488 \times 10^{-17}\ s^{-1}.$$

The activity or rate of disintegration at any instant, when the number of atoms is N, is

$$R = -\frac{dN}{dt} = \lambda N = (0.488 \times 10^{-17}\ s^{-1}) \times (2.53 \times 10^{21}) = \mathbf{1.23 \times 10^4\ s^{-1}}.$$

34) Prove that the instantaneous rate of change of the activity of a radioactive substance is inversely proportional to the square of its half life. [S.P.]

SOL: Activity or decay rate at any instant of time is $A = -N\lambda$
On differentiating equation (1) w.r.t. time, we get instantaneous rate of change of activity of radioactive substance.

$$\frac{dA}{dt} = -\frac{dN}{dt}\lambda$$

Again $\frac{dN}{dt} = -N\lambda$

Thus $\frac{dA}{dt} = N\lambda^2$

$$\frac{dA}{dt} = \frac{(0.693)^2 N}{T_{1/2}^2} \qquad \left(\because \lambda = \frac{0.693}{T_{1/2}}\right)$$

i.e. $\frac{dA}{dt} \propto \frac{1}{T_{1/2}^2}$

35) A radioactive sample contains 2.2 mg of pure ${}^{11}_{6}C$ which has half-life period of 1224 seconds. Calculate (i) the number of atoms present initially, (ii) the activity when 5μg of the sample will be left. (Avogadro's number is 6.023×10^{23}/mole).

SOL:

(i) 1 gram-atom of ${}_6C^{11}$ has a mass of 11 gram, and contains 6.023×10^{23} atoms (the Avogadro's number). Therefore, the number of atoms present (initially) in 2.2 mg $(= 2.2 \times 10^{-3}$ gram) of ${}_6C^{11}$ is

$$\frac{6.023 \times 10^{23}}{11\ g} \times (2.2 \times 10^{-3}\ g) = \mathbf{1.2046 \times 10^{20}}.$$

(ii) The number of atoms present in 5 μg $(= 5 \times 10^{-6}$ g) of the sample is

$$N = \frac{6.023 \times 10^{23}}{11\ g} \times (5 \times 10^{-6}\ g) = 2.7377 \times 10^{17}.$$

The decay constant is

$$\lambda = \frac{0.6931}{\text{half-life}} = \frac{0.6931}{1224\ s} = 5.662 \times 10^{-4}\ s^{-1}.$$

The activity of a radioactive sample at the instant when the sample has N atoms, is given by

$$R = \lambda N.$$

Putting the above values of λ and N, we have

$$R = (5.662 \times 10^{-4}\ s^{-1}) \times (2.7377 \times 10^{17})$$
$$= \mathbf{1.550 \times 10^{14}\ s^{-1}}.$$

1) **The nucleus $_{10}Ne^{23}$ decays by β -emission. Write down the decay equation and determine the maximum kinetic energy of the emitted electron. The atomic masses of $_{19}Ne^{23}$ and $_{11}Na^{23}$ are 22.994466 u and 22.989770.0 respectively.**

SOL:

The β -decay of $_{10}Ne^{23}$ is represented by the equation

$$_{10}Ne^{23} \longrightarrow {}_{11}Na^{23} + {}_{-1}\beta^{0} + \bar{\nu} + Q .$$

Ignoring masses of electron and anti-neutrino, the value of the liberated energy is given by

$$Q = [m\,({}_{10}Ne^{23}) - m\,({}_{11}Na^{23})]\,c^2$$
$$= [22{\cdot}994466\text{ u} - 22{\cdot}989770\text{ u}]\,c^2 = 0{\cdot}004696\text{ u} \times c^2.$$

Now, $1\text{ u} \times c^2 = 931{\cdot}5$ MeV.

$$\therefore \quad Q = 0{\cdot}004696 \times 931{\cdot}5 = \mathbf{4{\cdot}374\ MeV}.$$

This energy is shared by $_{11}Na^{23}$ nucleus and the electron-antineutrino pair. Since $_{11}Na^{23}$ is comparatively very much massive, almost whole of the liberated energy is taken by the electron-antineutrino pair. The electron will carry the maximum energy if the antineutrino carries no energy. Thus, the maximum kinetic energy of emitted electron is **4·374 MeV.**

36) **Define the terms 'decay constant' and 'half life' of a radioactive substance. Write their SI units. Establish relation between the two.**

37) **State the law of radioactive decay. Establish a mathematical relation between half-life period and disintegration constant of a radioactive nucleus.**

38) **What is meant by the activity of a radioactive substance? Show that the decay rate R of a sample of a radionuclide is related to the number of radioactive nuclei N at the same instant by the expression R =λ N.**

39) **The half life of $_{92}U^{238}$ against a-decay is 1.5×10^{17} s. What is the activity of a sample of $_{92}U^{238}$ having 25×10^{20} atoms.**

40) **State the law of radioactive decay. Establish a mathematical relation between half life period and disintegration constant of a radioactive nucleus. [Foreign 2007]**

41) **Define the terms half life period and decay constant of a radioactive substance. Write their SI units. Establish the relationship between the two. [All India 2006]**